



THE UNIVERSITY *of* EDINBURGH  
School of Mathematics

# Using specifications grading in a fully online course

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Joint with George Kinnear and Anna Wood

# Plan

About the course

Specifications grading

Outcomes



# Fundamentals of Algebra and Calculus

Developed with George Kinnear, see:  
Kinnear, G. (2019)

Delivering an online course using STACK

<http://doi.org/10.5281/zenodo.2565969>



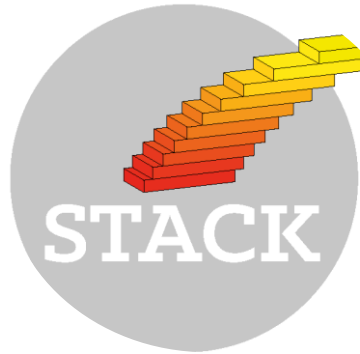
THE UNIVERSITY of EDINBURGH  
School of Mathematics

# Year 1 Curriculum

Semester 1	Semester 2
Introduction to Linear Algebra	Calculus and its Applications
Fundamentals of Algebra and Calculus	Proofs and Problem Solving
Option	Option



# Almost entirely online











# A typical week

◀ Week 3: Polynomials and rational functions

Week 5: Functions ▶

## Week 4: Principles of integration

-  Getting started
-  1. The area under a curve
-  2. Antiderivatives
-  3. Evaluating definite integrals
-  4. Finding areas
-  Week 4 Practice Quiz
-  Week 4 Final Test

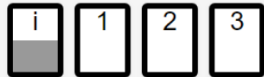
Your progress 



**Restricted** Not available unless: You achieve a required score in **Week 4 Practice Quiz**



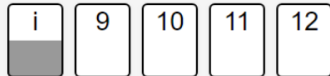
## INDEFINITE INTEGRALS



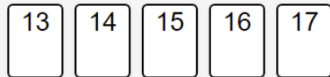
## STANDARD ANTIDERIVATIVES



## INTEGRATION BY INSPECTION



## MIXED PRACTICE



### Information

Flag question

### Antiderivatives and indefinite integrals

Remember that any function we can differentiate tells us about a corresponding antiderivative.

For example,  $\frac{d}{dx}(x^2) = 2x$  so we know  $x^2$  is an antiderivative of  $2x$ .

However, notice that we also have  $\frac{d}{dx}(x^2 + 1) = 2x$ . So  $x^2 + 1$  is also an antiderivative of  $2x$ .

In fact, if  $F(x)$  is an antiderivative of  $f(x)$  then so is  $F(x) + C$  where  $C$  is any constant. We can see this because differentiating both  $F(x)$  and  $F(x) + C$  gives  $f(x)$ .

We saw in the last section that antiderivatives are related to definite integrals:

#### Evaluation Theorem

If  $f(x) = G'(x)$  (i.e.  $G$  is an antiderivative of  $f$ ) then

$$\int_a^b f(x) dx = G(b) - G(a).$$

Because of this connection, we also talk about indefinite integrals:

The **indefinite integral**  $\int f(x) dx = F(x) + C$  means  $F'(x) = f(x)$ .

It represents the most general antiderivative of  $f$ , so must always include an arbitrary constant (usually  $+C$ ).

Note that:

- we say that  $f(x)$  is the **integrand**.
- The  $dx$  is very important because it indicates the variable we are integrating with respect to.
- the function  $F$  is often just referred to as the **integral** of  $f$ .

The notation is very similar for definite and indefinite integrals – the only difference is whether we attach limits to the integral sign. However, notice that the result of an indefinite integral is a *function*, whereas the definite integral gives a *number*.

#### Example

Returning to the example above, we can write the indefinite integral

$$\int 2x dx = x^2 + C$$

to represent the fact that  $x^2 + C$  is the most general antiderivative of  $2x$ .

### Question 1

Tries remaining: 1

Marked out of 1.00

Flag question

Which of the following are antiderivatives of  $x^6$ ?

- (a)  $\frac{x^7}{7} + 6$
- (b)  $\frac{x^7}{7} + 1$
- (c)  $\frac{x^7}{7}$
- (d)  $\frac{x^6}{7}$
- (e)  $6x^2 + x$
- (f)  $x^7$

Check

### Question 2

Tries remaining: 1

Marked out of 1.00

$\int x^2 dx =$

Check

Simple questions  
to check  
understanding



QUADRATICS

1 1 2 3 4

QUADRATIC INTERSECTIONS

1 5 6 7 8 9 1 10

CUBICS

1 1 11 1

HIGHER DEGREES

1 12 1 13 14 1 15

Finish attempt ...

Sketching graphs of cubics

Using the Factor Theorem, we can take the fully factorised form of a cubic and read off its roots. This enables us to make a sketch of the graph.

**Example**

Sketch the graph of the polynomial function  $f(x) = x^3 - 3x^2 + 4$ .

$f(x) = x^3 - 3x^2 + 4$

0:00 / 2:50

Video worked examples

Different forms of cubic graph are possible, based on the factors in the fully factorised form:

- each linear factor will give a root,
- any repeated linear factors will give rise to a repeated root on the graph,
- a quadratic factor which cannot be factorised further will mean the graph only has one real root.

Another thing to look for is the sign of the  $x^3$  coefficient - if it is positive, then the graph goes off to  $+\infty$  as  $x$  increases, while if it's negative the graph will go off to  $-\infty$ .

Information

Flag question

- The following are all the possible forms that the factorised cubic can take:
1.  $a(x - \alpha)(x - \beta)(x - \gamma)$
  2.  $a(x - \alpha)^2(x - \beta)$
  3.  $a(x - \alpha)^3$
  4.  $(x - \alpha)(ax^2 + bx + c)$  with  $b^2 - 4ac < 0$

Complete the following table showing what a sketch of the graph might look like for each form:

	$a(x - \alpha)(x - \beta)(x - \gamma)$	$a(x - \alpha)^2(x - \beta)$	$a(x - \alpha)^3$	$(x - \alpha)(ax^2 + bx + c)$ where $b^2 - 4ac < 0$
$a > 0$				
$a < 0$				

Matching/sorting activities



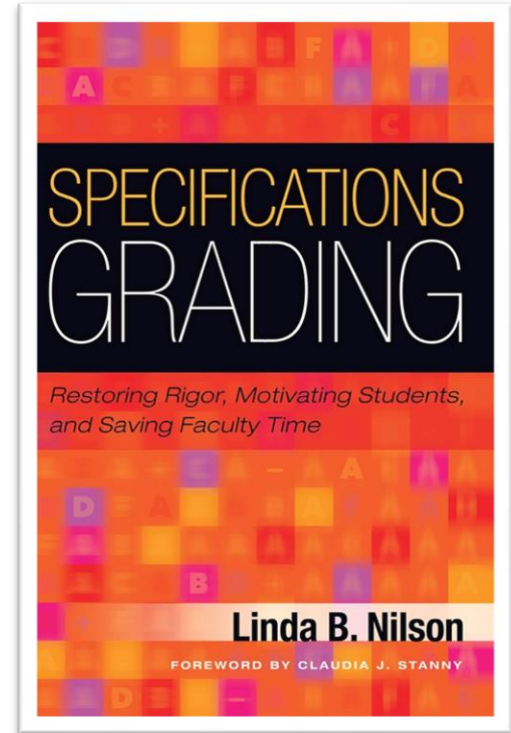


# Specifications grading



# Theory

- Individual assessments are graded pass/fail
- Some amount of resubmission is allowed
- Letter grades are based on performance across multiple assessments











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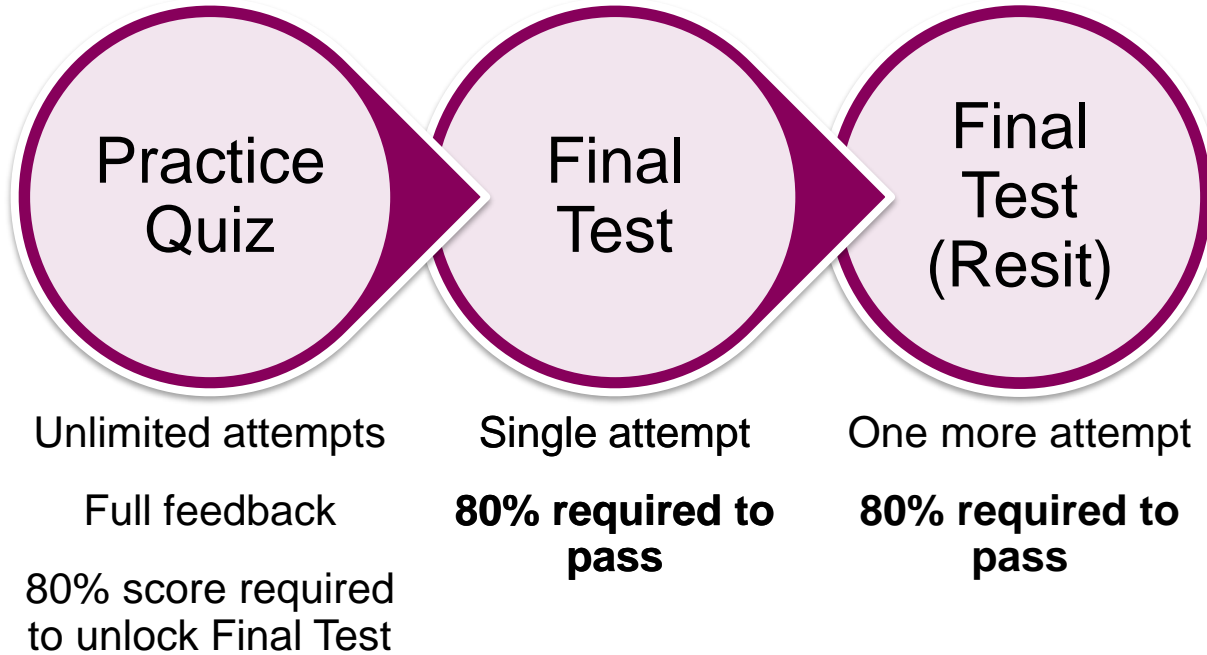
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# Implementation in FAC



Each week gets either a

**Mastery  
(80%+)**

or

**Distinction  
(95%+)**



# What grade will you get?

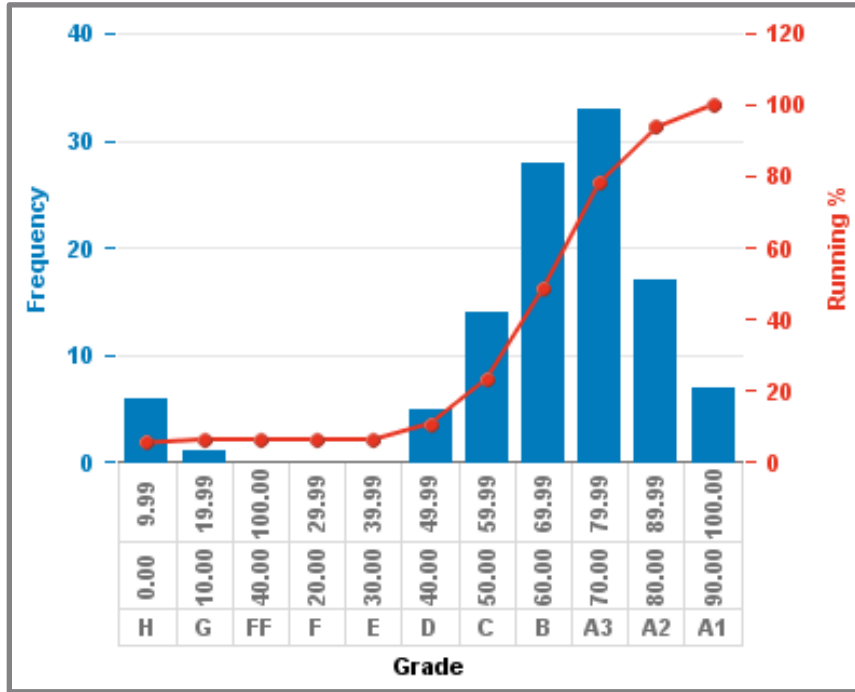
Number of units Mastered (80%+)	Number of Distinctions (95%+)	Percentage awarded for the Unit Score	Equivalent Grade
Less than 7	-	0	F
7	-	45%	D
8	2 or 3	55%	C
9	4 or 5	65%	B
10	6 or 7	75%	A3
10	8 or 9	85%	A2
10	10	100%	A1



# Outcomes



# Results (2018/19)

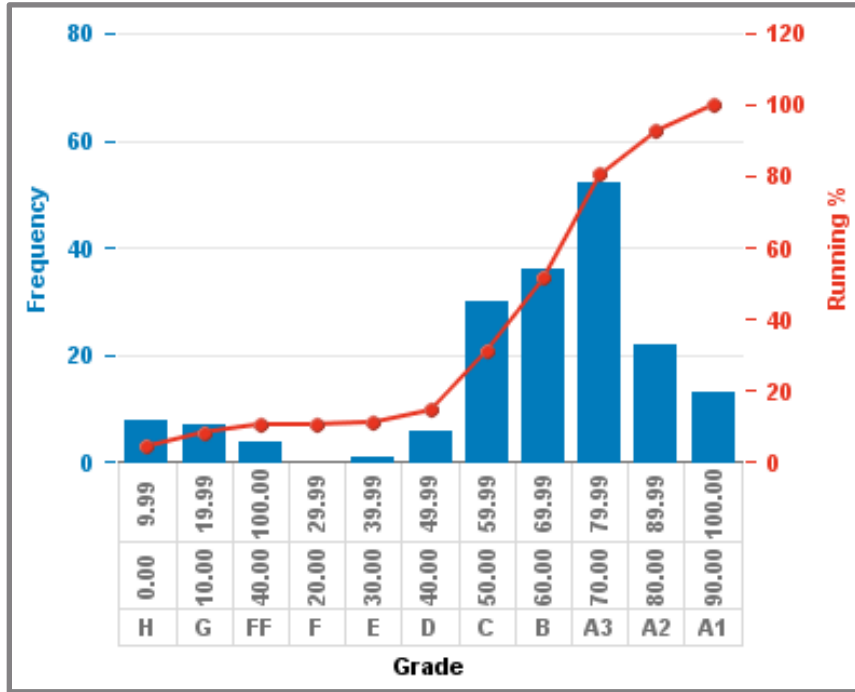


(N=113)

- Mean: 67
- Median: 70
- Pass rate: 94%



# Results (2019/20)



(N=181)

- Mean: 65
- Median: 69
- Pass rate: 88%





# Early failure

Number of units Mastered (80%+)	Number of Distinctions (95%+)	Percentage awarded for the Unit Score	Equivalent Grade
Less than 7	-	0	F
7	-	45%	D
8	2 or 3	55%	C
9	4 or 5	65%	B
10	6 or 7	75%	A3
10	8 or 9	85%	A2
10	10	100%	A1



# Mitigation

Lots of support

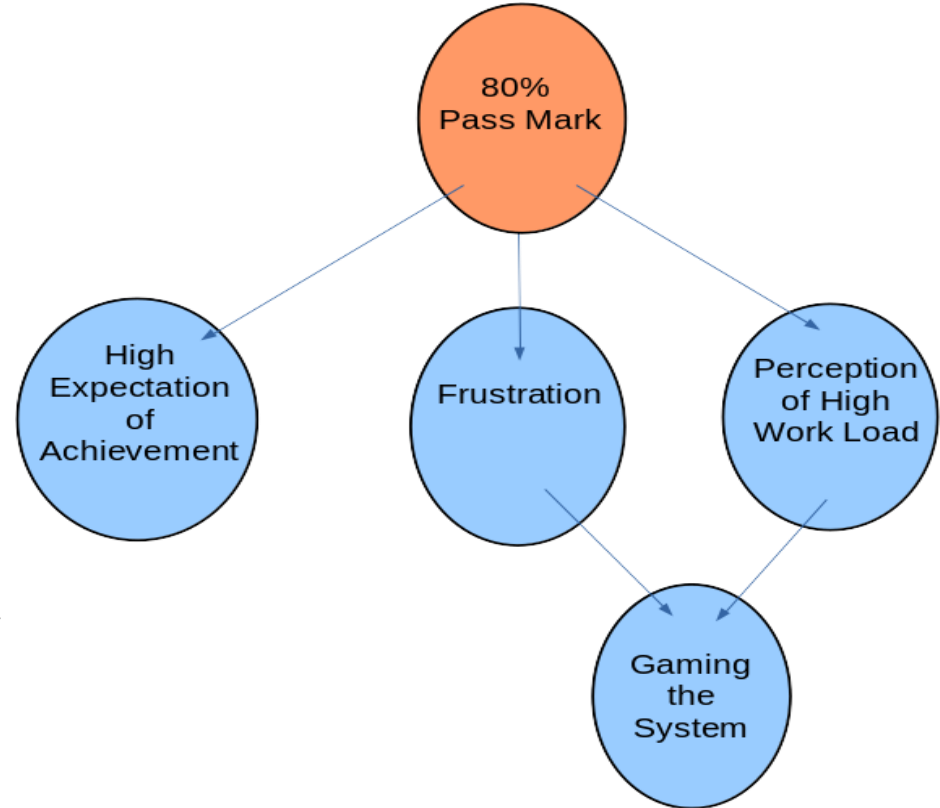
Engagement and progress monitoring

Opportunity for resubmission



# Student reaction

Mind-map of themes related to grading system



Gratwick, R., Kinnear, G., Wood, A. K., (2020)  
An online course promoting wider access to  
university mathematics. In Marks, R. (Ed),  
*Proceedings of the British Society for  
Research into Learning Mathematics*, 40 (1).



# Fewer marks per input

Tidy STACK question tool | Question tests & deployed variants

Given that  $\int_{-2}^2 f(x) dx = 9$ ,  $\int_{-2}^3 f(x) dx = 10$ ,  $\int_{-2}^2 g(x) dx = 14$ , and  $\int_2^3 g(x) dx = 4$ , which of the following can be evaluated?

(Enter the value if you can find it, otherwise enter **none** )

(a)  $\int_{-2}^3 f(x) - g(x) dx =$

(b)  $\int_{-2}^{-2} x^2 f(x) dx =$

(c)  $\int_{-2}^2 3f(x) + 5g(x) dx =$

(d)  $\int_2^3 f(x) dx =$

(e)  $\int_{-2}^2 x^2 f(x) dx =$

Check



# Partial credit

Fully factorise the polynomial  $p(x) = 3x^4 + 16x^3 + 3x^2 - 46x + 24$ , given that  $x = -3$  is a root.


$$p(x) = (x^2 + 2x - 3)(x + 4)(3x - 2)$$

Your last answer was interpreted as follows:

$$(x^2 + 2x - 3)(x + 4)(3x - 2)$$

The variables found in your answer were:  $[x]$

Check

 Your answer is partially correct.

Your answer is not factored. You could still do some more work on the term  $x^2 + 2x - 3$ .

The factor  $3x - 2$  is correct.

The factor  $x + 4$  is correct.

Marks for this submission: 0.50/1.00.



# Errors carried forward

Tidy STACK question tool | Question tests & deployed variants

Consider the function  $f(t) = 3t^2 - 6t + 8$ .

Let  $A(x)$  be the value of the area under the graph of  $y = f(t)$ , between  $t = x$  and  $t = x + 1$ .

Give expressions for  $A(x)$  and  $A'(x)$ . Your answers should be polynomials in terms of the variable  $x$ . Remember from unit 4 how to calculate the area under a graph between two points on the axis!

$A(x) =$

Your last answer was interpreted as follows:

$$x^2$$


The variables found in your answer were:  $[x]$

$A'(x) =$

Your last answer was interpreted as follows:

$$2x$$

The variables found in your answer were:  $[x]$

 Your answer is partially correct.  
Your expression for the area is incorrect.



# Conclusions

Encourages high standards

Can lead to frustration

Careful question-writing required

Promotes mastery of the subject



**Thank you!**



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