E-Assessment in Pure Mathematics

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Foundations and Proof

- First-year UK undergraduate pure mathematics module
- Emphasis on definitions, proofs and their techniques
- Approximately 300 to 350 students
- Assessed via a summative end-of-term exam
- Formative weekly written homework assignments which were encouraged but not compulsory

Motivation

- To help ease the transition between A-Level mathematics and university level pure mathematics by providing students with an extrinsic motivation to engage with learning material early on
- To emphasize the small details within proofs, such as defining notation, which are not necessarily emphasized in written homework

Implementation: Overview

- Summative as well as formative
- Open-book
- 48 hour deadline - later extended to one week
- Infinitely many attempts - submit once!
- Bi-weekly, alternating with written homework assignments

Implementation: Blackboard

Benefits of using Blackboard:

- Automatic marking
- Supports LaTeX and a variety of question types
- Allows for automatic feedback tailored to selected answer choices
- Feedback can be made available once deadline has passed
- Questions and answer choices can be randomised
- Easy to maintain once set up

Implementation: Blackboard

Drawbacks of using Blackboard:

- Very slow – setting up quizzes can take a long time!
- Does not support all LaTeX packages
- Individual feedback does not support LaTeX
- Display errors

Implementation: Quiz Questions

- Four 'easier' questions to familiarise students with definitions and to test knowledge
  - Multiple-choice questions
  - Multiple-answer questions
  - Matching questions
  - Fill-in-the-blank questions

Example: Multiple-Answer Question

Which of the following statements are true? Select **all** that apply.

- [ ] a. If p and q are prime and $p \mid q$, then $p = q$.
- [ ] b. We have that 1 is prime.
- [ ] c. If p and q are relatively prime, then both p and q are prime.
- [ ] d. There are infinitely many primes.
- [ ] e. Each $n \in \mathbb{N}$ has a unique prime factorisation.

Example: Matching Question

Let $i \in \mathbb{N}$ and define $A_i = \left\{2n : n \in \mathbb{N}, n \geq \frac{i}{2}\right\}$ and $B_i = \left\{2n-1 : n \in \mathbb{N}, n \geq \frac{i+1}{2}\right\}$. Please match the following equations.

- ▾ $\bigcup_{i \in \mathbb{N}} A_i$	A. $\mathbb{N}$
- ▾ $\bigcap_{i \in \mathbb{N}} B_i$	B. $2\mathbb{N} = \{2n : n \in \mathbb{N}\}$
- ▾ $\bigcup_{i \in \mathbb{N}} (A_i \cup B_i)$	C. $2\mathbb{N} - 1 = \{2n-1 : n \in \mathbb{N}\}$
	D. $\emptyset$

Example: Fill-in-the-blank Question

Please fill in the following blank:

Let $x \in \mathbb{Z}$ with $-17 \leq x \leq 51$. If $x \equiv 2 \pmod 5$ and $x \equiv 3 \pmod 7$, then $x =$

*Do **not** use spacing when entering your answer*

Implementation: Quiz Questions

Three ‘harder’ questions to emphasise the small details within proofs and to emphasise the structure of proofs

- Multiple-choice questions
- Matching questions

Implementation: Quiz Questions

Examples of students' common misconceptions when writing proofs:

- Not defining notation
- Working backwards in a proof
- Incorrect assumptions in a proof by contradiction
- Only proving one half of an 'if and only if' statement
- Incorrectly forming the contrapositive
- Using the wrong base case in a proof by induction
- Using regular induction instead of strong induction

Example: Multiple-Choice Question

Let $f = \{(x, x^4) : x \in \mathbb{R}\}$. Which one of the following proofs is correct?

- We will show that f is not a function.

  Let $x^4 = 16$. Then $x = 2$ or $x = -2$. Hence $f(2) = 16$ and $f(-2) = 16$. Therefore, f is not a function.

- We will show that f is a function.

  Suppose $x, y \in \mathbb{R}$ are such that $x^4 \neq y^4$. Taking roots, it follows that $x \neq \pm y$. Hence, for each $x \in \mathbb{R}$, we have that x^4 is unique. Therefore, f is a function.

- We will show that f is a function.

  Suppose $x, y \in \mathbb{R}$ are such that $x^4 \neq y^4$. For example, take $x^4 = 16$. Then $x = \pm 2$. Since $x^4 \neq y^4$, we have that $y^4 \neq 16$, so $y \neq \pm 2 = x$. Hence, y is unique for x. Therefore, f is a function.

- We will show that f is not a function.

  Suppose $x, y \in \mathbb{R}$ are such that $x \neq y$. Then $x^4 \neq y^4$ is not necessarily true since we could have $x = -y$. Hence, for each value x^4, we may have more than one solution for x. Therefore, f cannot be a function.

Example: Feedback on Correct Answer Choice

By Definition 1.5, we have that f is a function if for each $x \in \mathbb{R}$ we have that $f(x)$ is unique such that $(x, f(x)) \in f$. Here, we have that $f(x) = x^4$.

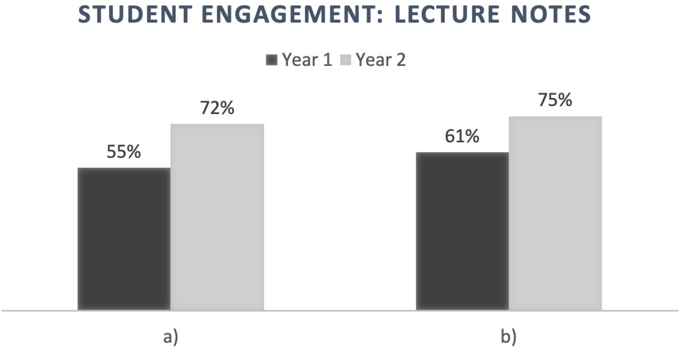
In order to argue that $f(x)$ is unique for each x, we consider two distinct outputs of f, $f(x)$ and $f(y)$ where $f(x) \neq f(y)$, and show that they must originate from two different inputs, x and y where $x \neq y$. This shows that two distinct outputs cannot originate from the same input, which makes $f(x)$ unique for each x.

Example: Feedback on Incorrect Answer Choice

Here, we chose $x^4 = 16$ and showed that there are two choices for x. This does not contradict that f is a function. Recall that by Definition 1.5, we have that f is a function if for each x there exists a unique $f(x)$ such that $(x, f(x)) \in f$, so the fact that two different x map to the same $f(x)$ is not a contradiction. For example, if $g(2) = 16$ and $g(2) = -16$, then g would not be a function. However, in our case we have $f(2) = 16$ and $f(-2) = 16$.

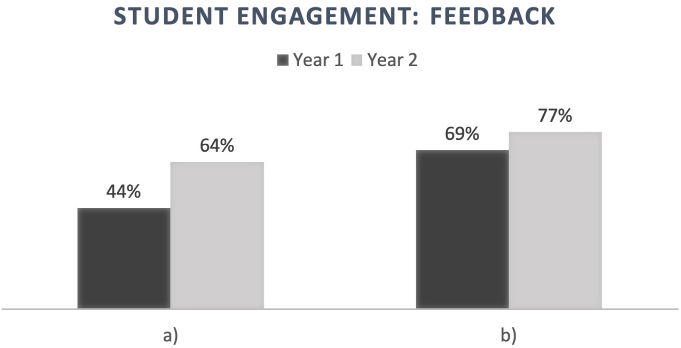
As a matter of fact, we have that f is indeed a function because if $f(x) = x^4 \neq y^4 = f(y)$ then $x \neq \pm y$. This shows that two distinct outputs cannot originate from the same input, which makes $f(x)$ unique for each x.

Student Survey Feedback: Engagement



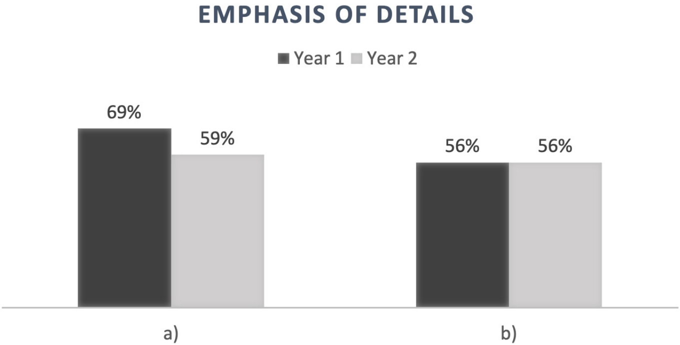
a) The online quizzes helped me to engage with the lectures notes early on
b) The online quizzes helped me to keep up with the material in the lecture notes

Student Survey Feedback: Engagement



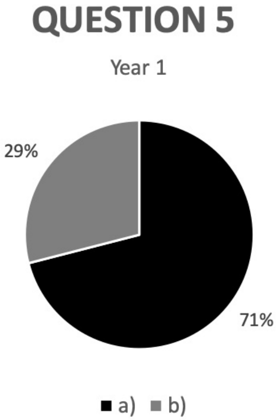
a) I read the available feedback
b) The feedback helped me to understand why my answer choice was incorrect

Student Survey Feedback: Emphasis of Details



a) The online quizzes helped me to pay attention to small details when writing proofs
b) The feedback helped me to pay attention to small details when writing proofs

Student Survey Feedback: Recommendation



a) Yes, I would recommend the use of online quizzes in other modules
b) No, I would not recommend the use of online quizzes in other modules

Conclusions

- Online quizzes successfully achieved their intended goals
- Great complementary tool to traditional written homework assignments
- While Blackboard is not flawlessly adapted to mathematics, its use for online quizzes is sustainable

Future Work

An e-learning platform for pure mathematics

- Focuses on proofs
- Can be used for assessment as well as formative exercises
- Assessment: pre-programmed proofs, which are easy to modify, and templates for setting up proofs from scratch
- Formative exercises: practise questions and step-by-step guides through proofs
- Automatic feedback: links to relevant definitions, results, and formative exercises

Any input or tips for sources of funding are welcome!
Please get in touch: s.zegowitz@gmail.com

Thank you!