Recent developments in the use of *maths e.g.*

– using e-assessment in the learning process

Martin Greenhow, Brunel University

Maths e.g. at Brunel University

- http://www.mathcentre.ac.uk:8081/mathseg/
- Questions database spans GCSE, A-level, undergraduate topics, ADULT LEARNERS/employability aptitude
- >~5000 'question spaces' in the database span MC, NI, RNI, PNI, TFU, MR, NI+confidence, Revealed MC, drag&drop etc. NEW a statement selection question SS (no free-form maths input … yet): PROS/CONS?
- About 1500 users take circa 30,000 tests p.a. Students from Maths, Economics, Electrical and Electronic Engineering, Computing, Foundations of Engineering, Foundations of IT, PGCE and Sports Science, even MSc student diagnostics. **Much commonality.** NEW BioMaths questions

Commonality – a European engineering syllabus

- **Core content**
- Set union, intersection, difference and complement. Existence and universal quantifiers. Direct and indirect proof, proof by induction.
- Definition of a function. Monotonicity of a function. Inverse function and combined function. Properties of basic functions. Hyperbolic functions and their inverses.
- Complex numbers and their basic properties (sum, difference, product, quotient, conjugate and modulus), presenting and calculating complex numbers both in coordinate form and polar form, complex roots.
- Limit and continuity of a function. One-sided and improper limits, l'Hospital's rule.
- Derivative as limit of difference quotient. Differentiating basic functions, products and quotients, chain rule. Studying the values and extrema of a function based on derivatives..
- Basics of integral calculus.

- **Complementary knowledge**
- Preimage, injection, surjection, bijection.
  Roots of real valued polynomials, factorisation.
- Sandwich principle. Intermediate value theorem, continuity of inverse function.
- Derivative of inverse function, linear approximations.
- Applications of integral, like area and volume.

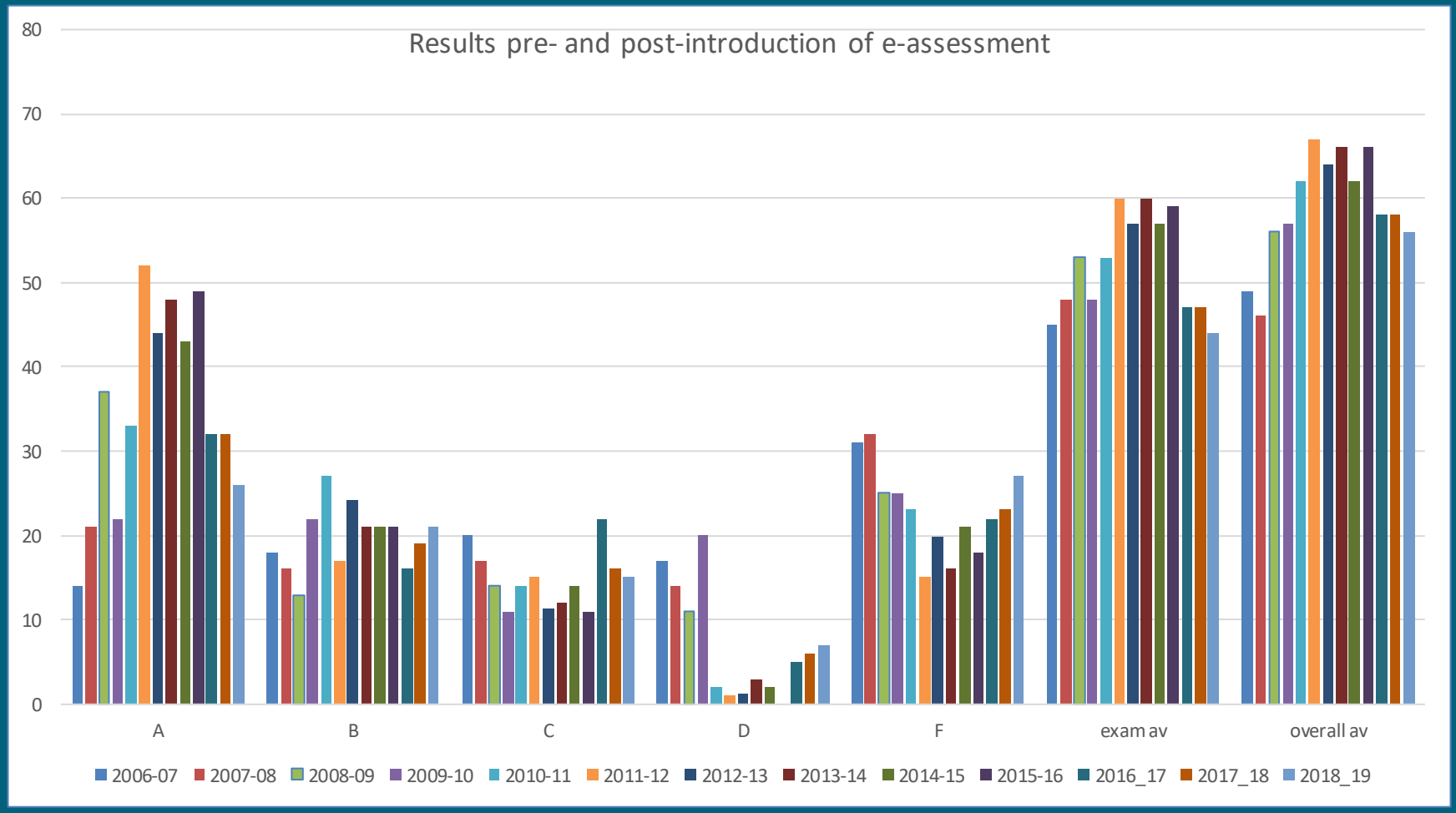
- **Specialist knowledge**
- Mean value theorem.

Done
Impossible
MG on the case

CAA in practice

- Low-stakes summative assessment - invigilated or not. Weekly or fortnightly tests. Pre and post tutorials but NO lectures.
- Preferred scheme: best-ever mark from their first 5 attempts counts towards their module mark; not invigilated and group work is allowed/encouraged. **Exam pass required!**
- **Mostly we aim to promote learning/confidence … we don't really need more marks!**
- **Benchmark against other students? Grand wizards.**

Maths for Economics year-on-year results (% in grade/% mark) circa 600,000 questions done



What happened next?

Covid -19!

CAA exams with randoms essential

Cheating (aliasing/using illegal software e.g. Wolfram alpha, Excel)

so upload their workings - do it!!!!!

The last two years' worth of bars on the chart are somewhat bonkers!

About half the students got an A … about the same number of F's as in previous years (some students are determined to fail!)

What should happen next?

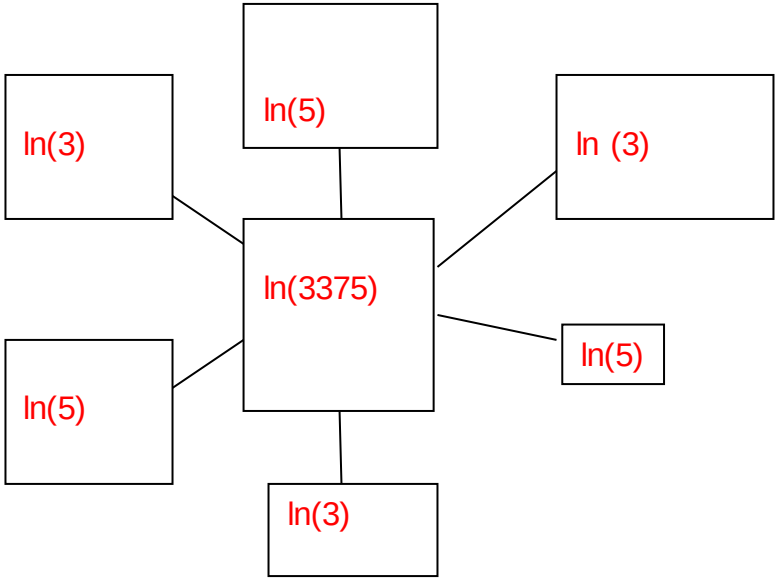
I retire next month (but *maths e.g.* will continue) so here's a couple of questions for everyone to consider so you can set better tests *a priori*:

How difficult is a question? Operation count (no), difficulty of concepts(?), question entropy (possible?)

How discriminating is a question? – no idea!

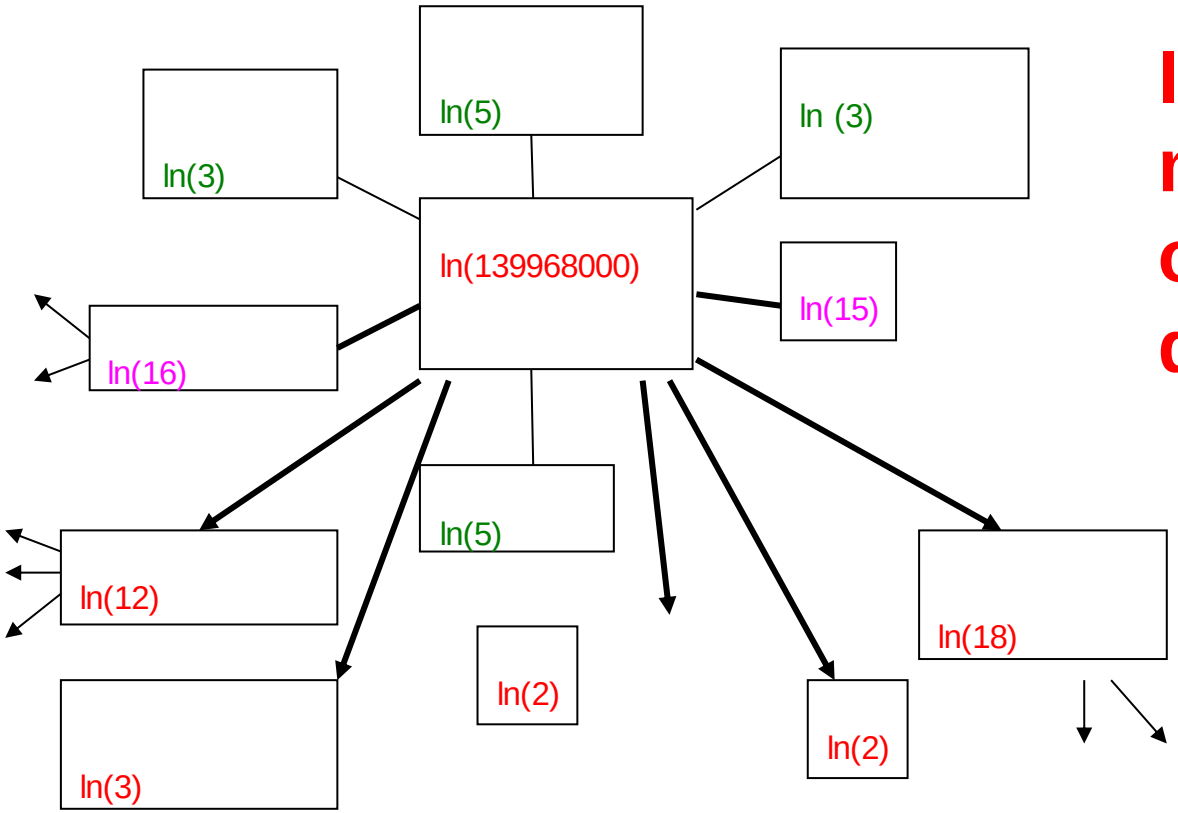
See **Effective computer-aided assessment of mathematics; principles, practice and results** *Teaching Mathematics and its Applications*, Volume 34, Issue 3, September 2015, https://doi.org/10.1093/teamat/hrv012

$$\int_2^4 x^3\,dx = \left[\frac{x^4}{4}\right]_2^4 = \frac{1}{4}\left(4^4 - 2^4\right) = \frac{1}{4}(256 - 16) = \frac{240}{4} = 60$$



More difficult if integration limits are negative and/or right -> left

$$\int_2^4 x^{-3/2}\,dx = -2\left[x^{-1/2}\right]_2^4 = -2\left(\frac{1}{4^{1/2}} - \frac{1}{2^{1/2}}\right) = -2\left(\frac{1}{2} - \frac{1}{\sqrt{2}}\right) = -1+\sqrt{2}$$



Is this a measure of difficulty?

Ratio of entropies = 2.3

What does this mean?
Rank ordering ok
(according to the data)

What should happen next?

A taxonomy of errors!

See **Effective computer-aided assessment of mathematics; principles, practice and results** *Teaching Mathematics and its Applications*, Volume 34, Issue 3, September 2015, https://doi.org/10.1093/teamat/hrv012

Print this screen | Colours & Fonts | Mathletics

Algebra\Algebraic fractions\Partial fractions\Irreducible factors\Quadratic & quadratic

Bruce is trying to decompose the following rational function into the sum of partial fractions, but he may have made a mistake.

If there is a mistake in Bruce's workings shown below, in which line was the mistake first made?

line 1: $\dfrac{10x^3-26x^2-8x-8}{-8x^4+2x^3-23x^2+12x-15}=\dfrac{10x^3-26x^2-8x-8}{(-2x^2-x-5)(4x^2-3x+3)}$

line 2: $\dfrac{10x^3-26x^2-8x-8}{-8x^4+2x^3-23x^2+12x-15}=\dfrac{Ax+B}{(-2x^2-x-5)}+\dfrac{Fx+G}{(4x^2-3x+3)}$

line 3: $10x^3-26x^2-8x-8=(Ax+B)(4x^2-3x+3)+(Fx+G)(-2x^2-x-5)$

line 4: $10 = 4A - 2F$

line 5: $-26 = -3A + 4B - F - 2G$

line 6: $-8 = 3A - 3B - 5F - G$

line 7: $-8 = 3B - 5G$

line 8: $\therefore A=5, B=-1, F=5 \text{ and } G=1$

line 9: $\dfrac{10x^3-26x^2-8x-8}{-8x^4+2x^3-23x^2+12x-15}=\dfrac{5x+1}{(-2x^2-x-5)}+\dfrac{5x-1}{(4x^2-3x+3)}$

Reverse engineering
But do we want them to do this?
Will they use Wolfram alpha? Yep!

Please input the line number where the mistake first occurs, or input 0 if there is no mistake.

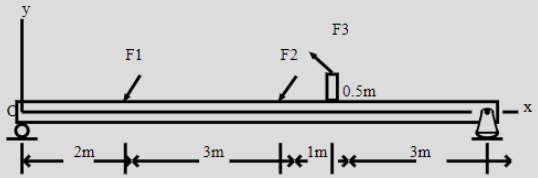
The error is in line ...

Maths e.g. - Google Chrome

Not secure | mathcentre.ac.uk:8081/mathseg/questionm.jsp?Question_ID=5337&tname=mechanics

A technician welds a small block onto a beam and then subjects this structure to the loads shown in the diagram below.

Replace the loading system by an equivalent resultant force and specify where the resultant's line of action intersects the beam measured from O.



$\mathbf{F}_1 = -34.41\mathbf{i} - 49.15\mathbf{j}, \mathbf{F}_2 = -87.42\mathbf{i} - 111.9\mathbf{j}, \mathbf{F}_3 = -220.71\mathbf{i} + 178.73\mathbf{j}.$

Give answers to 2 decimal places, but do not round until the end of your calculations!.

Resultant force: [ ] **i** + [ ] **j** N

Resultant's (acute) angle with the x axis: [ ] degrees

Distance from O: [ ] m

Feedback
same randomisation

Question - parameters
all randomised

Students mimic the feedback
in exam scripts

Maths e.g. - Google Chrome

Not secure | mathcentre.ac.uk:8081/mathseg/questionm.jsp?Question_ID=5337&tname=mechanics

To find the resultant force, we need to sum the horizontal (**i**), and vertical (**j**) components.

$\mathbf{F}_R = \mathbf{F}_1 + \mathbf{F}_2 + \mathbf{F}_3 = (\text{-}34.41\mathbf{i} + \text{-}49.15\mathbf{j}) + (\text{-}87.42\mathbf{i} + \text{-}111.9\mathbf{j}) + (\text{-}220.71\mathbf{i} + 178.7$

To find the angle with which the resultant force acts with the x-axis, we need to take the inverse of tan of the vertical force (**j**) devided by the horizontal force (**i**). Since we want the acute angle, we use the magnitude of the vertical and horizontal force, giving us:

$$\theta = \tan^{-1}\left(\frac{17.68}{342.55}\right) = 2.95$$

To find the distance at which the force acts from the origin, we need to take moments about the origin. Equating the moment from the original forces with the moment from the resultant force, we get:

$$\begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 2 & 0 & 0 \\ \text{-}34.41 & \text{-}49.15 & 0 \end{vmatrix} + \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 5 & 0 & 0 \\ \text{-}87.42 & \text{-}111.9 & 0 \end{vmatrix} + \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 6 & 0.5 & 0 \\ \text{-}220.71 & 178.73 & 0 \end{vmatrix} = (2 \times \text{-}49.15 +$$

$$17.68x = 524.93 \rightarrow x = \frac{524.93}{17.68} = 29.69\text{m}$$

Return to topics | New question

Preferable? TF(U) question

A positive quantity Q is known to depend on two positive variables as follows:

Mathematically we can write this as: $Q = k \frac{R}{S^3}$

The following claims are made.

*If you think the claim is **true, input T**.*
*If you think the claim is **false, input F**.*
*If you think the claim is **undecidable** on the basis of the information given, i*

Claim	T, F or U ?
If R decreases and S increases, Q decreases.	
If R decreases and S decreases, Q stays the same.	
If R decreases and S stays the same, Q increases.	
If R increases and S decreases, Q decreases.	

- Tests understanding (I think …)
- This question context (not fully shown) makes it clear that $k > 0$
- What if k, R or S < 0 ? Very much harder!

Question Manager

File Edit View Topic Question Tools Help

- Confidence appraisal
  - Algebra
  - Boolean algebra
  - Calculus
  - Functions
  - Linear algebra
  - Mechanics
  - Numbers
  - Statistics & probability
- Coordinate geometry
- Decision mathematics
  - Linear programming
    - Feasible regions
    - Optimisation (any method)
    - Simplex method
- Differentiation
  - Algebraic functions
  - Chain rule
  - Elementary functions
  - Implicit
  - Logarithmic
  - Optimisation - single variable
  - Parametric
  - Partial differentiation
  - Polynomials
  - Product rule
  - Quotient rule
  - Rational functions
  - Related rates
  - Series expansions

Description	Undergra...	A-level	GCSE	Difficulty	templa
Vectors; XC6	Level 1	FP3		Hard	
Vectors; XC6	Level 1	FP3		Interme...	
Matrices; XC6	Level 1	FP3		Easy	defaul
Matrices; XC6	Level 1	FP3		Interme...	defaul
Determinants; XC6	Level 1	FP3		Interme...	defaul

Press F1 for help

Confidence appraisal\Linear algebra

No Knowledge : No Confidence : Little Confidence : Some Confidence : Confident : Very Confident :

Topic	Your response
Given $C = \begin{bmatrix} 5 & 4 \\ 9 & 4 \\ -7 & 1 \\ 3 & -5 \end{bmatrix}$, find 5**A**.	
Given $A = \begin{bmatrix} 7 & 4 \\ 1 & -5 \end{bmatrix}$ and $B = \begin{bmatrix} -8 & -9 \\ -6 & 4 \end{bmatrix}$, find $\mathbf{A} - \mathbf{B}^T$.	
Given $A = \begin{bmatrix} 7 & 4 \\ 1 & -5 \end{bmatrix}$ and $C = \begin{bmatrix} 5 & 4 \\ 9 & 4 \\ -7 & 1 \\ 3 & -5 \end{bmatrix}$, find, if possible, **AC** and **CA**.	
Write $\begin{bmatrix} -5 & 2 & 3 \\ -1 & 1 & 2 \\ 2 & -3 & 2 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 1 \\ 5 \\ -2 \end{bmatrix}$ as three simultaneous equations.	
Write $\begin{bmatrix} 8 & 3 & 8 \\ -4 & -7 & 7 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 7 \\ -5 \end{bmatrix}$ as two simultaneous equations in x and y and solve for each in terms of z.	
Given $\begin{bmatrix} -5 & 2 & 3 \\ -1 & 1 & 2 \\ 2 & -3 & 2 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 1 \\ 5 \\ -2 \end{bmatrix}$, find z using Cramer's rule.	
Given **C** = **AB** where **A** is a given 5×7 matrix and **B** is a given 7×2 matrix, determine, if possible, $c_{3,2}$ and $c_{2,3}$.	
Given $Q = \begin{bmatrix} -5 & 2 & 3 \\ -1 & 1 & 2 \\ 2 & -3 & 2 \end{bmatrix}$, find tr(-3Q).	

Could use in breakout rooms?

Brunel University London Connect Assistant

Connect Assistant
Engine Not Running

Start Engine

Type here to search

15:15
20/05/2018

A statistics question - tables, tools or formula?

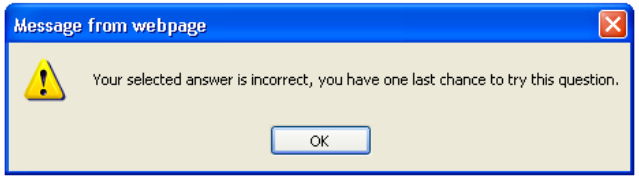
Statistics\Distributions\Binomial\Cumulative probability\Decontextual\by formula or 4dp tables

From the Cumulative Binomial distribution table below find and click the probability stated below, where X follows a binomial distribution and has the given value of n. The table below provides corresponding probabilities for ***X*** less than or equal to stated values of ***x***.

You can choose your answer from the Binomial distribution table by clicking your mouse. You will not be able to change your mind after you click on an answer, so make sure you choose your answers carefully before clicking. This question holds 2 marks, where full marks can be achieved by selecting the correct answer at the first attempt.

n= 20

Table



$P(X \leq 10 | p = 0.25)$ has the value of (click on the table)

P=	0.05	0.1	0.15	0.2	0.25	0.3	0.35	0.4	0.45	0.5
n=5,x=0	0.7738	0.5905	0.4437	0.3277	0.2373	0.1681	0.116	0.0778	0.0503	0.0312
1	0.9774	0.9185	0.8352	0.7373	0.6328	0.5282	0.4284	0.337	0.2562	0.1875
2	0.9988	0.9914	0.9734	0.9421	0.8965	0.8369	0.7648	0.6826	0.5931	0.5
3	1	0.9995	0.9978	0.9933	0.9844	0.9692	0.946	0.913	0.8688	0.8125
4	1	1	0.9999	0.9997	0.999	0.9976	0.9947	0.9898	0.9815	0.9688
n=10,x=0	0.5987	0.3487	0.1969	0.1074	0.0563	0.0282	0.0135	0.006	0.0025	0.0025
1	0.9139	0.7361	0.5443	0.3758	0.244	0.1493	0.086	0.0464	0.0233	0.0233
2	0.9885	0.9298	0.8202	0.6778	0.5256	0.3828	0.2616	0.1673	0.0996	0.0996
3	0.999	0.9872	0.95	0.8791	0.7759	0.6496	0.5138	0.3823	0.266	0.266

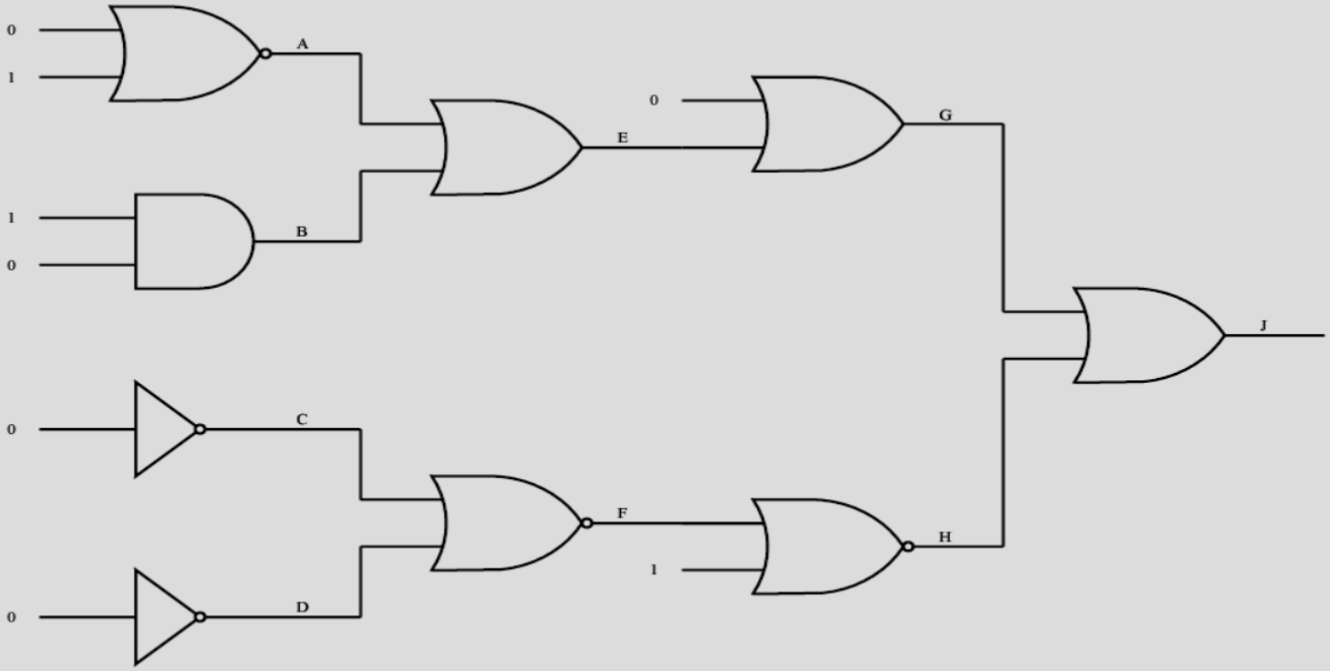
start | Shared Documents | Presentation.ppt [Co... | Question Manager | EN | 20:28

A logic question – more straightforward?

Boolean algebra\Logic\Gates

1 of 1

Consider the following circuit:



Type here to search

A new question type

In the following the *leading entry* in a row is the left most non-zero entry in that row.

Which of the following properties **must** a matrix have to be in row echelon form:

1. Each leading entry of a row is in a column to the left of the leading entry of the row above it.
2. All entries in a column below a leading entry are zero.
3. The number of columns must be less than or equal to the number of rows.
4. All entries in a column below a leading entry are smaller than the leading e
5. All non-zero rows are above any rows of all zeros.
6. Each leading entry of a row is in a column to the right of the leading entry o
7. Each leading 1 is the only non-zero entry in its column.

$A =$

Important:You must select all properties that apply to the question and nothing

Input your answer string in the form of a sequence of increasing numbers e.g.

If you think none of the above properties apply, input *none*.

In the following the *leading entry* in a row is the left most non-zero entry in that row.

Which of the following properties **must** a matrix have to be in row echelon form:

1. The absolute value of all entries in a column below a leading entry are smaller than absolute value of the leading entry.
2. The matrix must be square.
3. Each leading entry of a row is in a column to the right of the leading entry of the row above it.
4. All non-zero rows are above any rows of all zeros.
5. Each leading entry of a row is in a column to the left of the leading entry of the row above it.
6. The leading entry in each non-zero row is 1.
7. The number of rows must be less than or equal to the number of columns.
8. Each leading entry of a row is in the same column or a column to the right of the leading entry of the row above it.
9. All non-zero rows are below any rows of all zeros.
10. All entries in a column below a leading entry are non-zero.
11. Each leading 1 is the only non-zero entry in its column.
12. The number of columns must be less than or equal to the number of rows.
13. All entries in a column below a leading entry are smaller than the leading entry.

$A =$

Important:You must select all properties that apply to the question and nothing else.

In the question test, MUST can be replaced by 'may', 'must not'

Not been used yet but coding is done

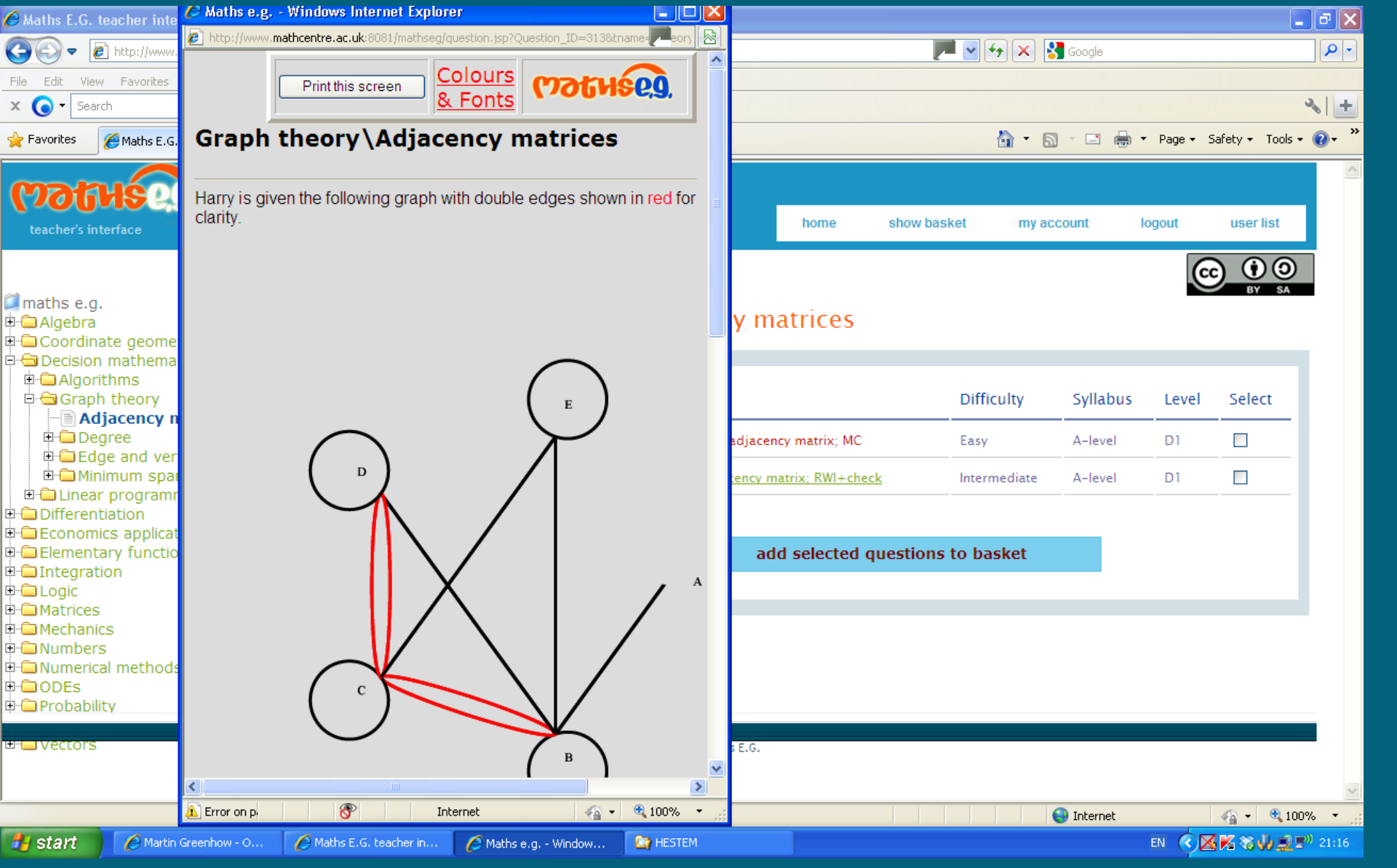
```
corrects_ref = new Array("","All non-zero rows are above any rows of all zeros.",
 "Each leading entry of a row is in a column to the right of the leading entry of the row above it.",
 "All entries in a column below a leading entry are zero.") …
```

definition of row echelon form

```
wrongs_ref = new Array("","The leading entry in each non-zero row is 1.",
 "Each leading 1 is the only non-zero entry in its column.",
"All non-zero rows are below any rows of all zeros.“,
 "Each leading entry of a row is in a column to the left of the leading entry of the row above it.",
 "Each leading entry of a row is in the same column or a column to the right of the leading entry of the
 row above it.",
 "All entries in a column below a leading entry are non-zero.",
 "All entries in a column below a leading entry are smaller than the leading entry.",
 "The absolute value of all entries in a column below a leading entry are smaller than absolute value
 of the leading entry.",
"The matrix must be square.",
"The number of rows must be less than or equal to the number of columns.",
"The number of columns must be less than or equal to the number of rows.");
```

So all that’s needed is teaching experience!

Teacher interface – shop on Amazon?



- Javascript, MathML and SVG provide a rich environment for setting objective questions
- Positive effects on students' perceptions and on exam performances
- Widely applicable database of questions
- Good source of reverse-engineered questions for all, especially teachers
- Maths e.g. Try it at:

http://www.mathcentre.ac.uk:8081/mathseg/

http://www.mathcentre.ac.uk:8081/mathsegteacher/

works on all browsers, PC & Mac, I-pads and smart phones, no link to VLEs yet!

Where are we going with all this?

Can we get systems to share or work together? Maybe not! But you can still 'steal' what is useful and view the source code too.
Future-proofing? Yes, so far!

Maths e.g.'s legacy may be more as a learning resource (certainly how students use the feedback)

What we can all do right now (this afternoon!)

Link individual questions to any other learning material that supports links e.g. add ‘Try one yourself’ button to open question/feedback in a new window.

Maths e.g. - Google Chrome
Not secure | http://www.mathcentre.ac.uk:8081/mathseg/questionm.jsp?Question_ID=1&tname=algebra
Select Language
Print this screen
Colours & Fonts
mathseg

Algebra/Adding polynomials

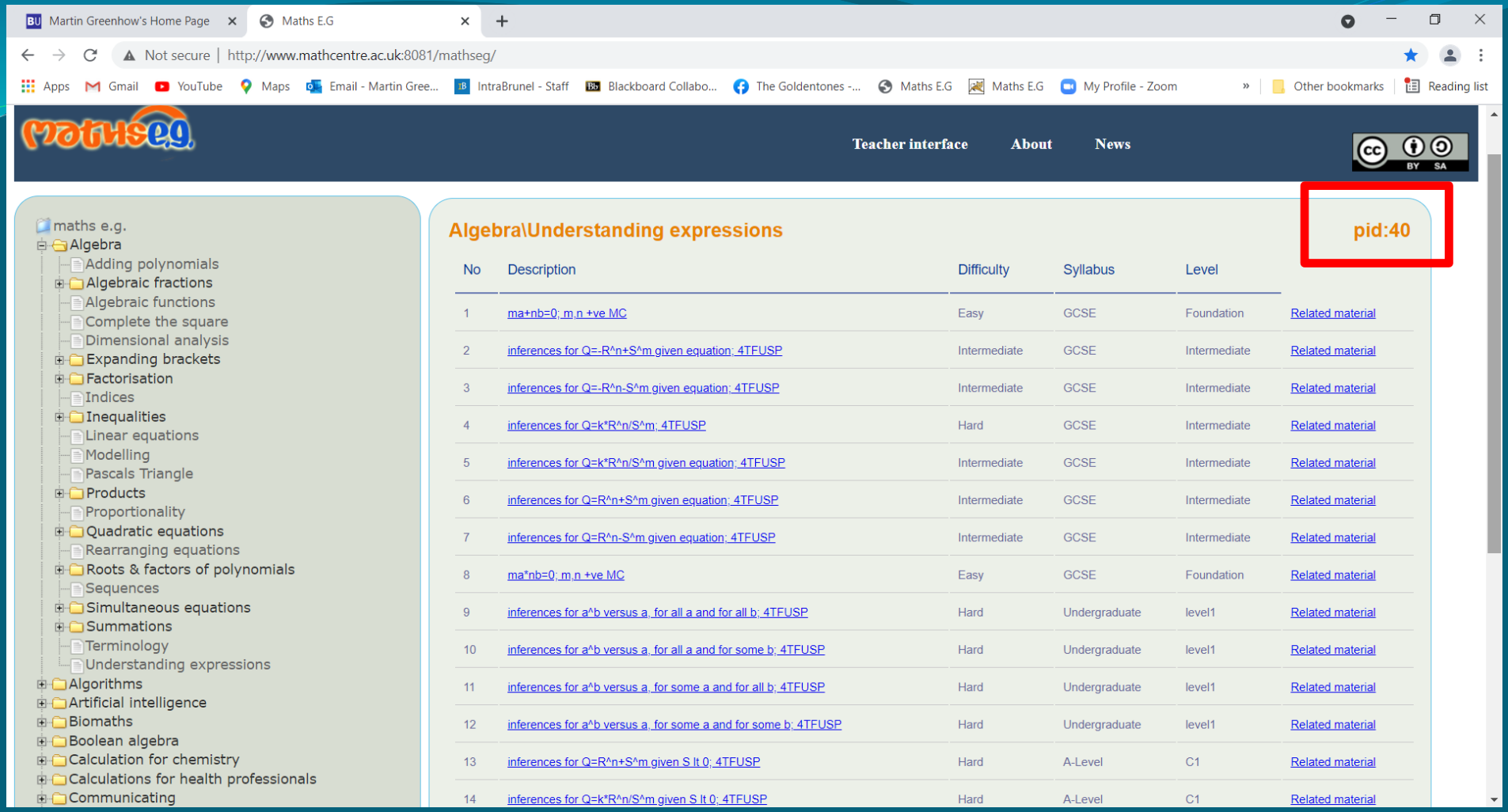
Simplify as much as possible

$$\left[-7 - x + 2x^2 + x^3 - 6x^4\right] + \left[5 - 2x + 4x^2 + 6x^3 - 9x^4 + 2x^5\right]$$

- $-2 + x + 6x^2 + 7x^3 + 3x^4 + 2x^5$
- $-2 - 3x + 6x^2 + 7x^3 - 15x^4 + 2x^5$
- $-35 + 9x - 16x^2 - 45x^3 + 33x^4 + 23x^5$
- $-12 + x - 2x^2 - 5x^3 + 3x^4 - 2x^5$

What we can also do right now (this afternoon!)

Link individual TOPICs to any other learning material that supports links e.g. add ‘Try one yourself’ button to open all questions in a topic in new window



http://www.mathcentre.ac.uk:8081/mathseg/topic.jsp?pid=40

Or link to whole tests you have created - CHECK, CHECK, CHECK! (VLE issue)
… but I am not going to provide tests since I'd never include what you want and also issues of privacy (so no *Mathematics for Dogs and Cats*)

Finally take what you want, including code,
… and tell me what's missing (no promises!)
Let's share.

Good luck!

martin.greenhow@brunel.ac.uk
martin.greenhow1@outlook.com