Analysis of students' answer process based on STACK answer data

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This work was supported by JSPS KAKENHI Grant Number 21H04412

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Introduction

One of the most important features of LMS is online assessment, specifically, the computer-aided assessment (CAA)

- CAA can assess students' answers as correct or incorrect automatically and instantaneously
- Traditional question types: true-or-false, multiple-choice, fill-in numeric, etc

Mathematics e-Learning systems can provide questions on calculation, in which mathematical expressions with an algebraic style are automatically assessed

- STACK, Numbas, WeBWorK, Mobius, etc…
- Partial score is possible when an expression only satisfies some of the required properties

In order to give partial score, automatic categorisation of incorrect answers is required.

- We proposed a prototype of STACK's PRT (Potential Response Tree) for automatic categorisation of solutions to questions involving derivatives of functions of the type $(ax^m + b)(cx^n + d)$
  - doi:10.5281/zenodo.5036454

Objectives

We would like to analyse the process of answering a question, i.e., what kind of incorrect answers students went through before arriving at a correct answer, by using the function of categorising incorrect answers

Kurihara et al. represented the answer process of a student as a directed graph and visualised the overall tendency of the answer process (see later)

However, there is a drawback that detailed temporal information is unknown

We compensate for this by analysing the tendency towards correct answers by visualising the time variation of the types of incorrect answers

- We also want to investigate how feedback influences the students' answer process

Answering process of students (Kurihara et al.)

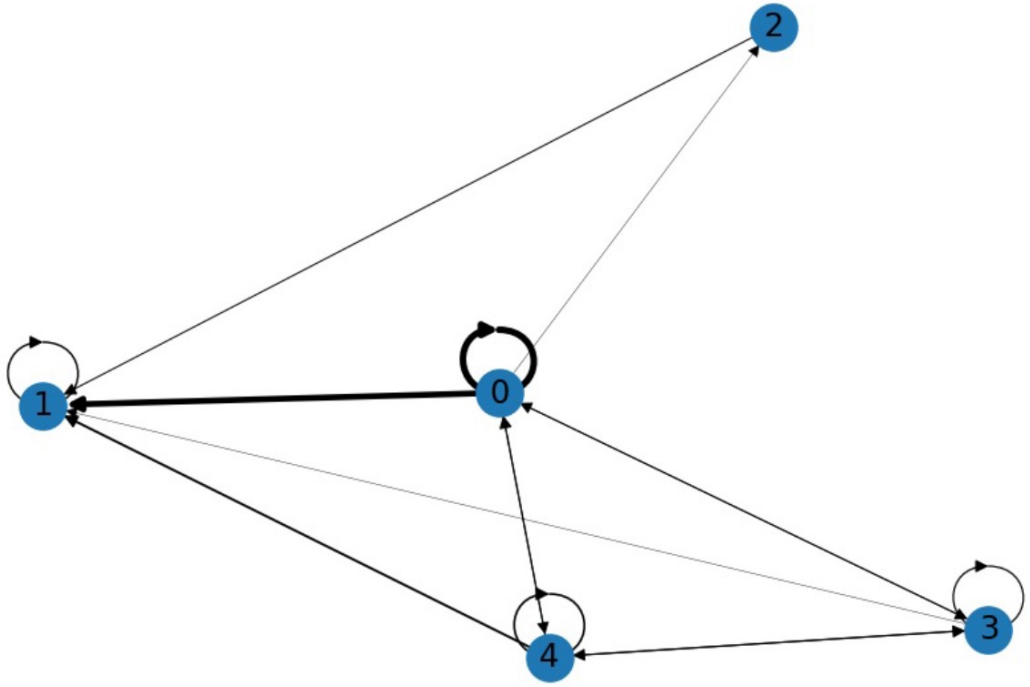
Calculate the following integrals. Do not forget the arbitrary constants.
次の積分を計算せよ。任意定数を忘れないこと。

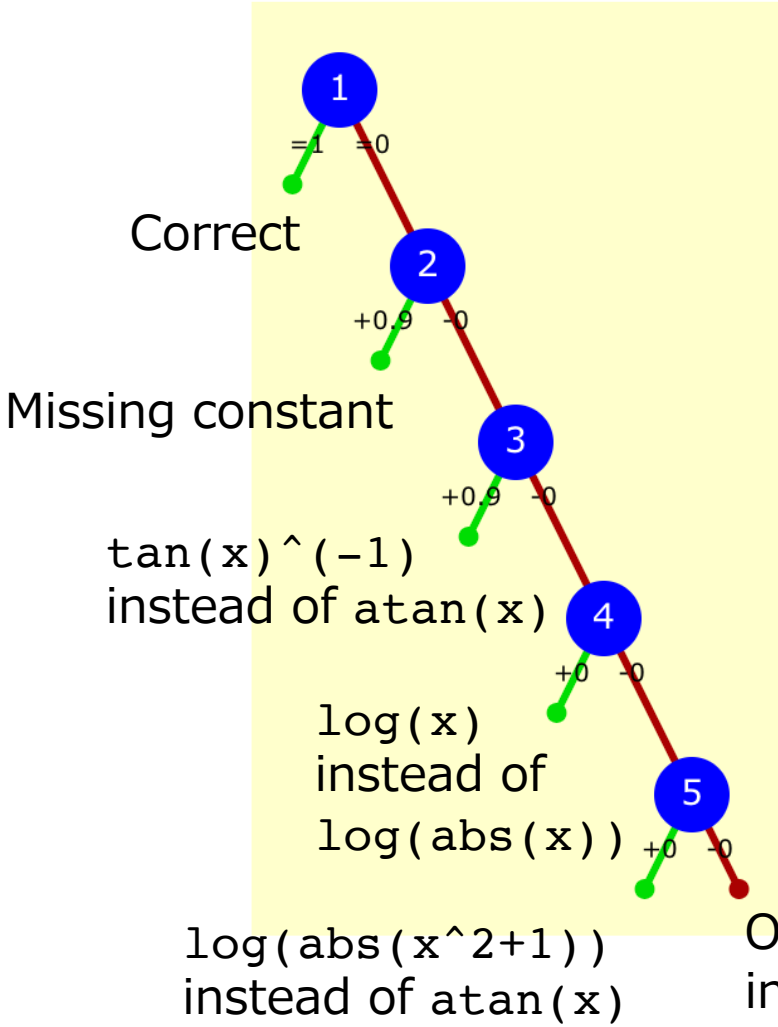
Tidy STACK question tool | Question is missing tests or variants.

$$\int \frac{x^3 + x + 1}{x^4 + x^2} \mathrm{d}x$$

Answer: $\log|x| - \arctan x - \frac{1}{x} + C$

`log(abs(x))-atan(x)-1/x+C`





(Kurihara and Nakamura, unpublished)

Answering process of students (Kurihara et al.)

Pros and cons of visualisation of answering process by using the directed graph

Pros

- It is possible to identify overall trends in the solution process
- It is possible to give tips on improving the PRT
  - There are many self-loops at node 0 → Improved PRT to provide more detailed categorisation of incorrect answers

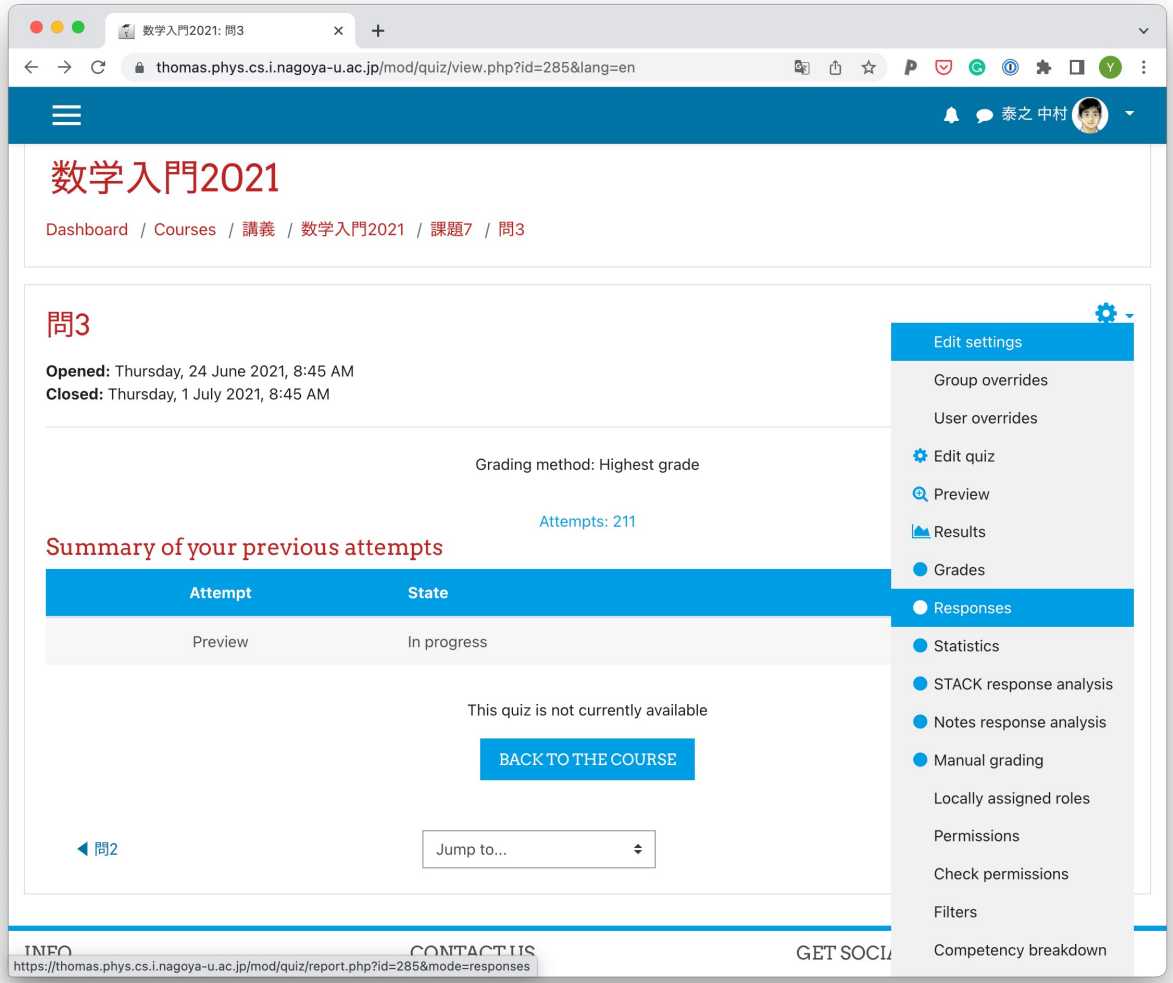
Cons

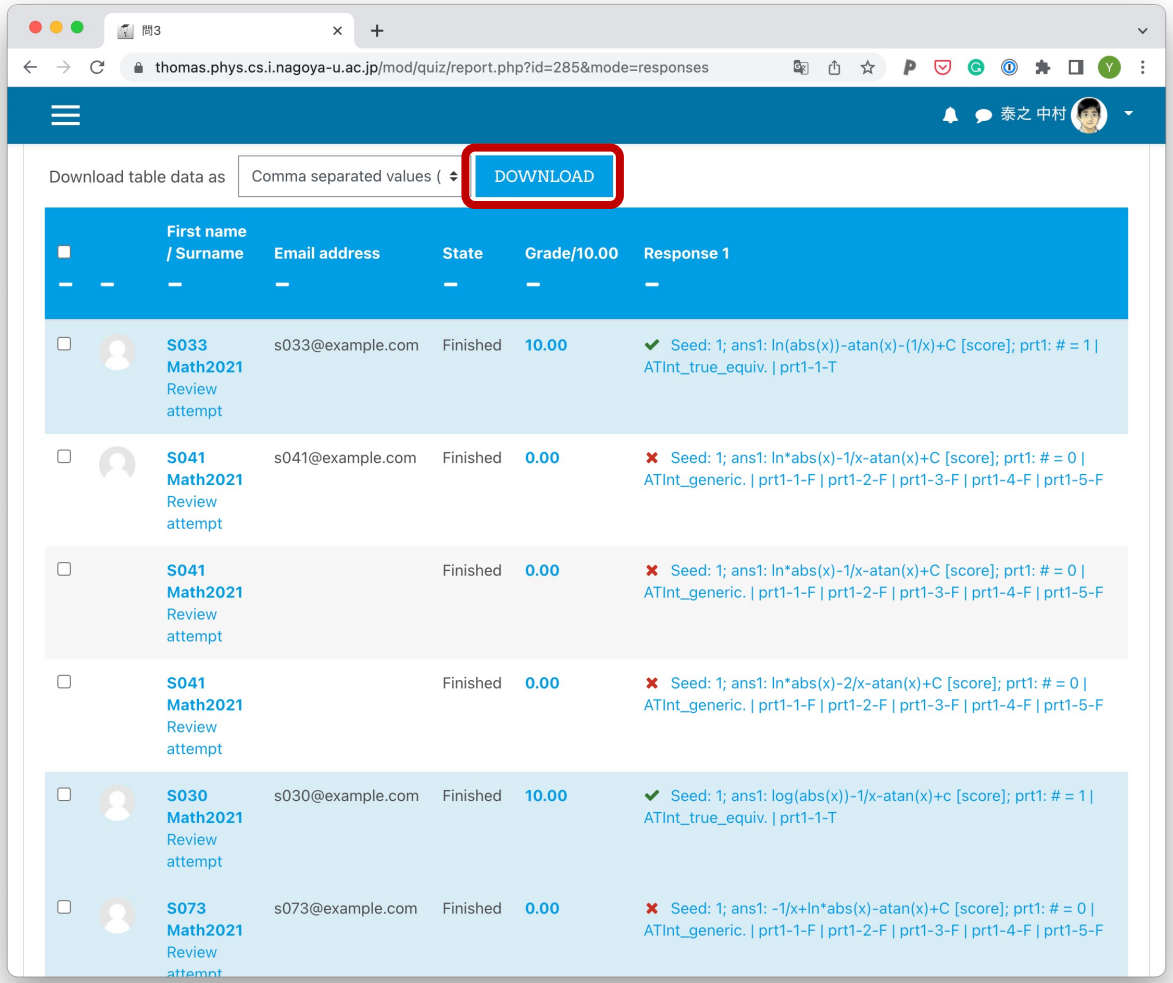
- Detailed temporal information of answering process of each student is unknown
- It is not clear how feedbacks influence the students' answering process

Objective: We compensate for the cons by analysing the tendency towards correct answers by visualizing the time variation of the types of incorrect answers

Data to be analysed

- 94 students
- Introduction of Mathematics (Liberal Arts Subjects) 2021





Data to be analysed

	A	B	C	D	E	F	G	H	I
1	Surname	First name	Email address	State	Started on	Completed	Time taken	Grade/10.00	Response 1
2	Math2021	S033	s033@example.com	Finished	2021/6/24 9:54	2021/6/24 20:16	10 hours 21 mins	1	Seed: 1; ans1: ln(abs(x))-atan(x)-(1/x)+C [score]; prt1: # = 1 | ATInt_true_equiv. | prt1-1-T
3	Math2021	S041	s041@example.com	Finished	2021/6/24 10:13	2021/6/24 10:14	1 min 46 secs		Seed: 1; ans1: ln*abs(x)-1/x-atan(x)+C [score]; prt1: # = 0 | ATInt_generic. | prt1-1-F | prt1-2-F | prt1-3-F | prt1-4-F | prt1-5-F
4	Math2021	S041	s041@example.com	Finished	2021/6/24 10:15	2021/6/24 10:16	1 min 40 secs		Seed: 1; ans1: ln*abs(x)-1/x-atan(x)+C [score]; prt1: # = 0 | ATInt_generic. | prt1-1-F | prt1-2-F | prt1-3-F | prt1-4-F | prt1-5-F
5	Math2021	S041	s041@example.com	Finished	2021/6/24 10:44	2021/6/24 11:28	43 mins 28 secs		Seed: 1; ans1: ln*abs(x)-2/x-atan(x)+C [score]; prt1: # = 0 | ATInt_generic. | prt1-1-F | prt1-2-F | prt1-3-F | prt1-4-F | prt1-5-F
6	Math2021	S030	s030@example.com	Finished	2021/6/24 11:12	2021/6/24 16:22	5 hours 9 mins	1	Seed: 1; ans1: log(abs(x))-1/x-atan(x)+c [score]; prt1: # = 1 | ATInt_true_equiv. | prt1-1-T
7	Math2021	S073	s073@example.com	Finished	2021/6/24 11:52	2021/6/24 12:00	7 mins 12 secs		Seed: 1; ans1: -1/x+ln*abs(x)-atan(x)+C [score]; prt1: # = 0 | ATInt_generic. | prt1-1-F | prt1-2-F | prt1-3-F | prt1-4-F | prt1-5-F
8	Math2021	S073	s073@example.com	Finished	2021/6/24 12:01	2021/6/24 12:01	12 secs		Seed: 1; ans1: -1/x+ln(x)-atan(x)+C [score]; prt1: # = 0 | ATInt_EqFormalDiff. ATInt_logabs. | prt1-1-F | prt1-2-F | prt1-3-F | prt1-4-
9	Math2021	S041	s041@example.com	Finished	2021/6/24 12:30	2021/6/24 12:30	13 secs		Seed: 1; ans1: ln*abs(x)-1/x-atan(x)+C [score]; prt1: # = 0 | ATInt_generic. | prt1-1-F | prt1-2-F | prt1-3-F | prt1-4-F | prt1-5-F
10	Math2021	S019	s019@example.com	Finished	2021/6/24 15:14	2021/6/29 1:03	4 days 9 hours		Seed: 1; ans1: log(x)+atan(x)+1/x+c [score]; prt1: # = 0 | ATInt_generic. ATInt_logabs. | prt1-1-F | prt1-2-F | prt1-3-F | prt1-4-F | pr
11	Math2021	S022	s022@example.com	Finished	2021/6/24 15:52	2021/6/24 16:04	12 mins 36 secs		Seed: 1; ans1: -1/x+ln(x)-atan(x)+C [score]; prt1: # = 0 | ATInt_EqFormalDiff. ATInt_logabs. | prt1-1-F | prt1-2-F | prt1-3-F | prt1-4-
12	Math2021	S022	s022@example.com	Finished	2021/6/24 16:05	2021/6/24 16:05	18 secs		Seed: 1; ans1: -1/x+ln(x)-atan(abs(x))+C [score]; prt1: # = 0 | ATInt_generic. ATInt_logabs. | prt1-1-F | prt1-2-F | prt1-3-F | prt1-4-F
13	Math2021	S022	s022@example.com	Finished	2021/6/24 16:05	2021/6/24 16:06	53 secs	1	Seed: 1; ans1: -1/x+ln(abs(x))-atan(x)+C [score]; prt1: # = 1 | ATInt_true_equiv. | prt1-1-T
14	Math2021	S045	s045@example.com	Finished	2021/6/24 20:23	2021/6/25 15:27	19 hours 3 mins	1	Seed: 1; ans1: log(abs(x))-1/x-atan(x)+C [score]; prt1: # = 1 | ATInt_true_equiv. | prt1-1-T
15	Math2021	S012	s012@example.com	Finished	2021/6/24 23:02	2021/6/27 14:54	2 days 15 hours		Seed: 1; ans1: ln*abs(x)-1/x-atan(x)+C [score]; prt1: # = 0 | ATInt_generic. | prt1-1-F | prt1-2-F | prt1-3-F | prt1-4-F | prt1-5-F
16	Math2021	S056	s056@example.com	Finished	2021/6/24 23:52	2021/6/30 12:31	5 days 12 hours	1	Seed: 1; ans1: ln(abs(x))-1/x-atan(x)+C [score]; prt1: # = 1 | ATInt_true_equiv. | prt1-1-T
17	Math2021	S009	s009@example.com	Finished	2021/6/25 8:39	2021/6/25 8:46	6 mins 55 secs	1	Seed: 1; ans1: log(abs(x))-1/x-atan(x)+C [score]; prt1: # = 1 | ATInt_true_equiv. | prt1-1-T
18	Math2021	S100	s100@example.com	Finished	2021/6/25 9:50	2021/6/25 9:52	2 mins 12 secs		Seed: 1; ans1: (((x*ln(x))-x*atan(x)-1)/x)+C [score]; prt1: # = 0 | ATInt_EqFormalDiff. ATInt_logabs. | prt1-1-F | prt1-2-F | prt1-3-F |
19	Math2021	S100	s100@example.com	Finished	2021/6/25 9:52	2021/6/25 9:53	37 secs		Seed: 1; ans1: ((x*log(x)-x*atan(x)-1)/x)+C [score]; prt1: # = 0 | ATInt_EqFormalDiff. ATInt_logabs. | prt1-1-F | prt1-2-F | prt1-3-F |
20	Math2021	S100	s100@example.com	Finished	2021/6/25 10:22	2021/6/25 10:40	18 mins 5 secs		Seed: 1; ans1: (x*log(x)-x*atan(x)-1)/x [score]; prt1: # = 0 | ATInt_EqFormalDiff. ATInt_logabs. | prt1-1-F | prt1-2-F | prt1-3-F | prt1-
21	Math2021	S100	s100@example.com	Finished	2021/6/25 10:40	2021/6/25 11:41	1 hour		Seed: 1; ans1: ((x*log(x)-x*atan(x)-1)/x)+C [score]; prt1: # = 0 | ATInt_EqFormalDiff. ATInt_logabs. | prt1-1-F | prt1-2-F | prt1-3-F |
22	Math2021	S078	s078@example.com	Finished	2021/6/25 11:07	2021/6/25 11:25	17 mins 44 secs	1	Seed: 1; ans1: ln(abs(x))-atan(x)-1/x+C [score]; prt1: # = 1 | ATInt_true_equiv. | prt1-1-T
23	Math2021	S100	s100@example.com	Finished	2021/6/25 11:41	2021/6/25 17:37	5 hours 56 mins		Seed: 1; ans1: ((-1/(x+2))+3/(x-1))+C [score]; prt1: # = 0 | ATInt_generic. | prt1-1-F | prt1-2-F | prt1-3-F | prt1-4-F | prt1-5-F
24	Math2021	S059	s059@example.com	Finished	2021/6/25 12:59	2021/6/25 13:17	18 mins 1 sec		Seed: 1; ans1: 1/2*ln(abs(x^2))-1/x-atan(x)+C [score]; prt1: # = 0 | ATInt_EqFormalDiff. | prt1-1-F | prt1-2-F | prt1-3-F | prt1-4-T
25	Math2021	S059	s059@example.com	Finished	2021/6/25 13:17	2021/6/25 13:19	1 min 23 secs	1	Seed: 1; ans1: ln(abs(x))-1/x-atan(x)+C [score]; prt1: # = 1 | ATInt_true_equiv. | prt1-1-T
26	Math2021	S073	s073@example.com	Finished	2021/6/25 13:37	2021/6/30 19:22	5 days 5 hours		Seed: 1; ans1: -1/x+ln(x)-atan(x)+C [score]; prt1: # = 0 | ATInt_EqFormalDiff. ATInt_logabs. | prt1-1-F | prt1-2-F | prt1-3-F | prt1-4-
27	Math2021	S100	s100@example.com	Finished	2021/6/25 17:40	2021/6/25 17:40	7 secs		Seed: 1; ans1: (1/x)+(1/x^2)-1/((x^2)+1)+C [score]; prt1: # = 0 | ATInt_generic. | prt1-1-F | prt1-2-F | prt1-3-F | prt1-4-F | prt1-5-F
28	Math2021	S090	s090@example.com	Finished	2021/6/25 20:31	2021/6/25 20:52	20 mins 45 secs	1	Seed: 1; ans1: log(abs(x))-1/x-atan(x)+C [score]; prt1: # = 1 | ATInt_true_equiv. | prt1-1-T
29	Math2021	S008	s008@example.com	Finished	2021/6/25 21:39	2021/6/25 21:43	3 mins 31 secs	1	Seed: 1; ans1: log(abs(x))-1/x-atan(x)+C [score]; prt1: # = 1 | ATInt_true_equiv. | prt1-1-T
30	Math2021	S074	s074@example.com	Finished	2021/6/26 10:02	2021/6/26 10:05	2 mins 6 secs		Seed: 1; ans1: ln(x)-(x*atan(x)+1)/x+C [score]; prt1: # = 0 | ATInt_EqFormalDiff. ATInt_logabs. | prt1-1-F | prt1-2-F | prt1-3-F | prt1-
31	Math2021	S074	s074@example.com	Finished	2021/6/26 10:05	2021/6/26 10:06	34 secs		Seed: 1; ans1: (x*ln(x)-x*atan(x)-1)/x+C [score]; prt1: # = 0 | ATInt_EqFormalDiff. ATInt_logabs. | prt1-1-F | prt1-2-F | prt1-3-F | prt
32	Math2021	S074	s074@example.com	Finished	2021/6/26 10:06	2021/6/26 10:12	5 mins 51 secs		Seed: 1; ans1: ln(x)+(-x*atan(x)-1)/x+C [score]; prt1: # = 0 | ATInt_EqFormalDiff. ATInt_logabs. | prt1-1-F | prt1-2-F | prt1-3-F | prt1

Analysis method

Question

- $\int \frac{x^2+x+1}{x^4+x^2}\,dx = \int \left(\frac{1}{x} - \frac{1}{x^2+1} + \frac{1}{x^2}\right) dx = \log|x| - \arctan x - \frac{1}{x} + C$

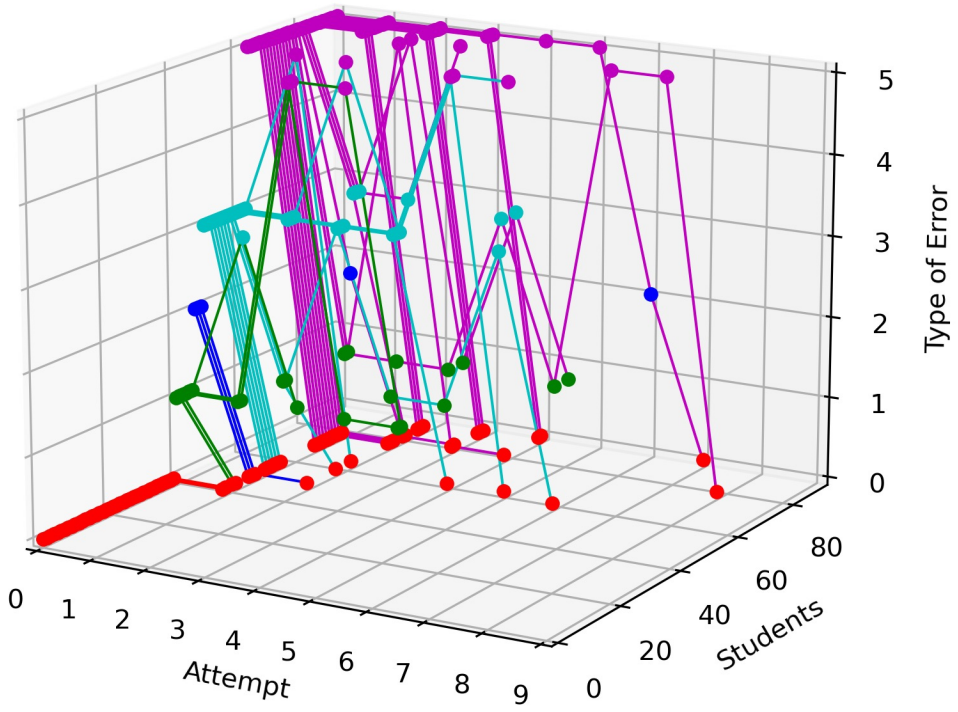
Solutions are grouped into six categories

- Correct answer (0) : `log(abs(x))-atan(x)-1/x+C`
- `tan(x)^(-1)` instead of `atan(x)` (1)
- Missing arbitrary constant (2)
- `log(x)` instead of `log(abs(x))` (3)
- `log(x^2+1)` instead of `atan(x)` (4)
- Other (5)
- The larger the number in brackets is, the greater the degree of error becomes

We visualise the solution process for each student

- Visualisation of how each student’s incorrect answer leads to the correct answer

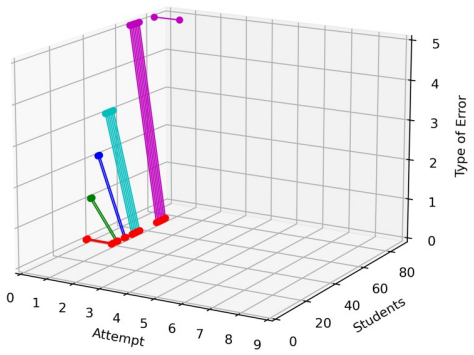
Visualisation of answering process



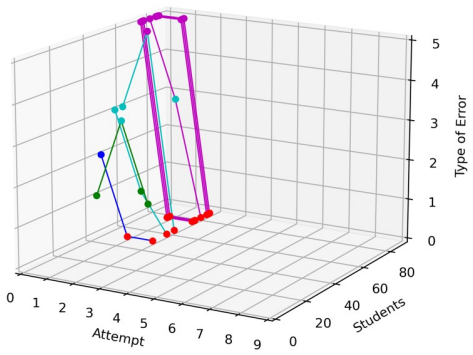
Coloured dots indicate the type of error

Coloured lines indicate which error type the answering process started with

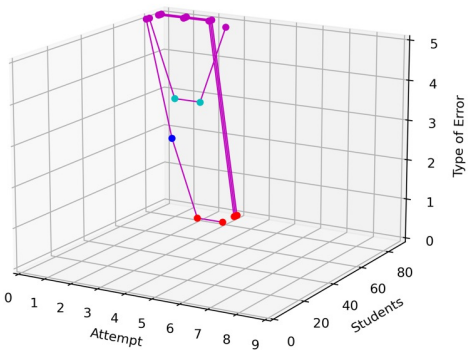
of attempts to reach a correct answer



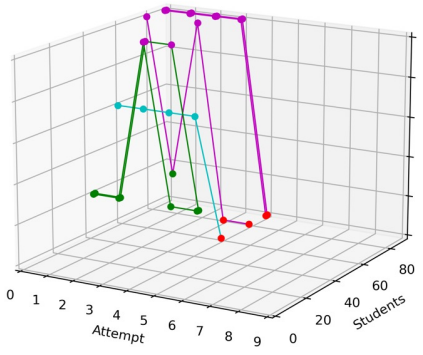
2 attempts



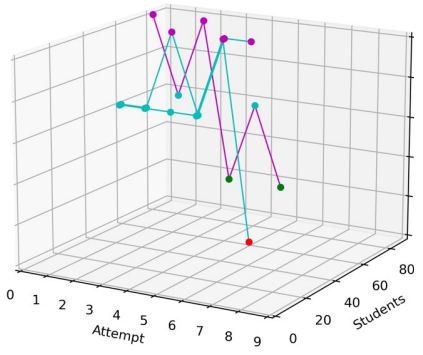
3 attempts



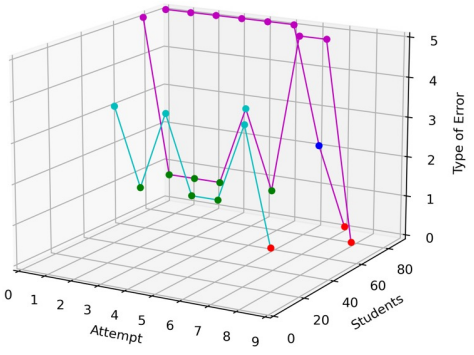
4 attempts



5 attempts



6 attempts



7-9 attempts

Summary

We analysed the solution process on the subject of integral questions involving three types of integrals

- $\int \frac{x^2+x+1}{x^4+x^2}\,dx = \int \left(\frac{1}{x} - \frac{1}{x^2+1} + \frac{1}{x^2}\right) dx = \log|x| - \arctan x - \frac{1}{x} + C$

We proposed a method for visualising how students go through incorrect answers to get to the correct answers

- 3D plot with three axes: number of attempts, student number and type of incorrect answer
- In the case of more than five attempts, we could see how students were trying (getting closer to or further away from the correct answer)
- Appropriate feedback may help students to understand

Future plan

- We study the effect of how feedback intervenes in the improvement of students' understanding in mathematics online tests