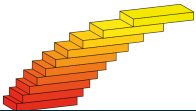
Developing interactive online workbooks for the mathematical education of general STEM students

Dr Konstantina Zerva
Dr David Quinn

School of Mathematics
The University of Edinburgh



Outline

1. Online assessment with STACK
2. Underlying pedagogy
3. Resources for Engineers: HELM workbooks
4. HELM workbooks $\rightarrow$ STACK workbooks
5. Use of workbooks in Engineering / Chemistry students

Necessity of automated assessment

Assessment sits at the heart of the learning process[1] *!*

A key driver to students' activity.

[1] Timmis et all. 2016

Necessity of automated assessment

Assessment sits at the heart of the learning process[1] *!*

A key driver to students' activity.

Purposes:

- To aid learning

[1] Timmis et all. 2016

Necessity of automated assessment

Assessment sits at the heart of the learning process[1] *!*

A key driver to students' activity.

Purposes:

- To aid learning
- High stakes, e.g. exams

[1] Timmis et all. 2016

Necessity of automated assessment

Assessment sits at the heart of the learning process[1] *!*

A key driver to students' activity.

Purposes:

- To aid learning
- High stakes, e.g. exams

Well designed assessments assure quality of the course.

[1] Timmis et all. 2016

Necessity of automated assessment

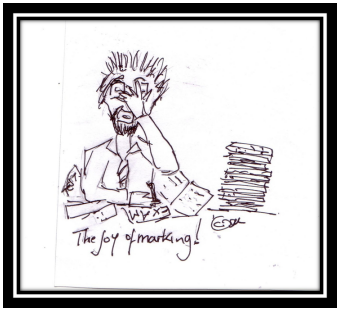
Assessment sits at the heart of the learning process[1]*!*

A key driver to students' activity.

Purposes:

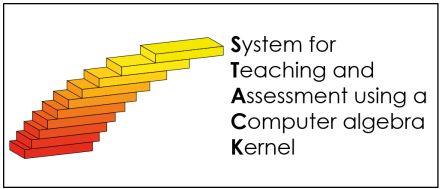
- To aid learning
- High stakes, e.g. exams

Well designed assessments assure quality of the course.

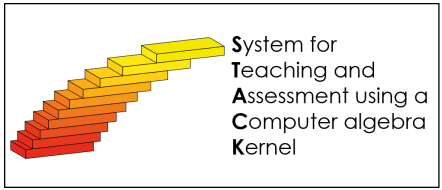


[1] Timmis et all. 2016

What is STACK?



What is STACK?



- Assessment system for Mathematics and Sciences.
- Optional plugin for Moodle Quiz (and ILIAS system).
- Uses the Computer Algebra System (CAS) Maxima.
- Students can type an answer at the form of an algebraic expression.
- Moves assessments well beyond multiple choice questions.

What is special about STACK – Design choices

Teachers should be able to write their own questions.

What is special about STACK – Design choices

Teachers should be able to write their own questions.

STACK separates **“validity”** and **“correctness”**.

What is special about STACK – Design choices

Teachers should be able to write their own questions.

STACK separates **“validity”** and **“correctness”**.

Multi-part questions should be possible.

What is special about STACK – Design choices

Teachers should be able to write their own questions.

STACK separates **"validity"** and **"correctness"**.

Multi-part questions should be possible.

STACK is rich in features:

What is special about STACK – Design choices

Teachers should be able to write their own questions.

STACK separates **"validity"** and **"correctness"**.

Multi-part questions should be possible.

STACK is rich in features:

- Questions can be randomised.
- Many different kinds of inputs.
- Partial credit is possible.

What is special about STACK – Design choices

Teachers should be able to write their own questions.

STACK separates **"validity"** and **"correctness"**.

Multi-part questions should be possible.

STACK is rich in features:

- Questions can be randomised.
- Many different kinds of inputs.
- Partial credit is possible.
- Plots can be dynamically generated.
- Line by line reasoning by equivalence.
- Full support for scientific units.

STACK Demonstration site

Demonstration site: https://stack-demo.maths.ed.ac.uk/demo/

Three lead developers:

- Chris Sangwin, The University of Edinburgh, UK
- Tim Hunt, (Moodle quiz lead developer) OU, UK
- Matti Harjula, Aalto, Finland.

Online assessment tools

We have good tools for online assessment of mathematics.

How can we use those tools effectively?

Assessment combined with resources

Putting the book inside the quiz

Assessment combined with resources

Putting the book inside the quiz

All materials are inside the quiz.

- Written notes
- Video clips (split up as needed)
- Online assessments

Assessment combined with resources

Putting the book inside the quiz

All materials are inside the quiz.

- Written notes
- Video clips (split up as needed)
- Online assessments

We started this in 2018, not just as a COVID response.

Fundamentals of Algebra and Calculus [2]

[2] Kinnear,G.: Delivering an online course using STACK

Fundamentals of Algebra and Calculus [2]

- The course is delivered fully online.

[2] Kinnear,G.: Delivering an online course using STACK

Fundamentals of Algebra and Calculus [2]

- The course is delivered fully online.
- Designed to help students with a wide range of backgrounds.

[2] Kinnear,G.: Delivering an online course using STACK

Fundamentals of Algebra and Calculus [2]

- The course is delivered fully online.
- Designed to help students with a wide range of backgrounds.
- **Course structure:** have the book inside the quiz.
  1. Textbook-style exposition;
  2. videos of worked examples;
  3. interactive applets;
  4. practise questions.

  Quizzes are the organising structure to promote engagement.

[2] Kinnear,G.: Delivering an online course using STACK

Fundamentals of Algebra and Calculus [2]

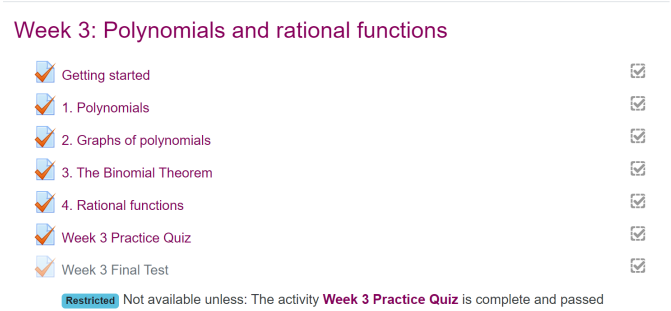
- The course is delivered fully online.
- Designed to help students with a wide range of backgrounds.
- **Course structure:** have the book inside the quiz.
  1. Textbook-style exposition;
  2. videos of worked examples;
  3. interactive applets;
  4. practise questions.

  Quizzes are the organising structure to promote engagement.
- **Mastery learning:** students need to score $\geq 80\%$ to pass each week's assessment.

[2] Kinnear,G.: Delivering an online course using STACK

Fundamentals of Algebra and Calculus

https://stack-demo.maths.ed.ac.uk/demo/course/view.php?id=4



Resources for Engineers

The mathematical education of engineers and scientists is important!

Our COVID-19 response to support STEM education.

HELM: Help Engineers Learn Mathematics

https://nucinkis-lab.cc.ic.ac.uk/HELM/helm_workbooks.html

HELM: Help Engineers Learn Mathematics

https://nucinkis-lab.cc.ic.ac.uk/HELM/helm_workbooks.html

HELM background:

- October 2002 – September 2005.
- £250,000 grant from the Higher Education Funding Council for England.
- Teams of writers and developers at six universities.

Many design decisions were carefully made!

Project goal

Combine FAC and HELM

Project goal

Combine FAC and HELM

- Comprehensive workbooks for engineers and scientists.

Project goal

Combine FAC and HELM

- Comprehensive workbooks for engineers and scientists.
- Based on coherent and tested resources
  (we obtained permission to use HELM from Loughborough).

Project goal

Combine FAC and HELM

- Comprehensive workbooks for engineers and scientists.
- Based on coherent and tested resources
  (we obtained permission to use HELM from Loughborough).
- Individual PDF files a manageable size, but big enough to
  be worthwhile.

Project approach

Interns and staff primarily at:

- The University of Edinburgh
- Loughborough University

Project approach

Interns and staff primarily at:

- The University of Edinburgh
- Loughborough University

Part of ASID project: Adapt, Support, Implement, Deliver

Project approach

Project approach

- Convert HELM source to Moodle (Mathjax)

Project approach

- Convert HELM source to Moodle (Mathjax)
- Implement questions within STACK where possible

Project approach

- Convert HELM source to Moodle (Mathjax)
- Implement questions within STACK where possible
  - sensible randomisation
  - author written solutions
  - PRT, deployed variants, question notes

Project approach

- Convert HELM source to Moodle (Mathjax)
- Implement questions within STACK where possible
  - sensible randomisation
  - author written solutions
  - PRT, deployed variants, question notes
- Follow strict style guide and authoring guidelines

Project approach

- Convert HELM source to Moodle (Mathjax)
- Implement questions within STACK where possible
  - sensible randomisation
  - author written solutions
  - PRT, deployed variants, question notes
- Follow strict style guide and authoring guidelines
- Work reviewed by local STACK expert

Project approach

- Convert HELM source to Moodle (Mathjax)
- Implement questions within STACK where possible
  - sensible randomisation
  - author written solutions
  - PRT, deployed variants, question notes
- Follow strict style guide and authoring guidelines
- Work reviewed by local STACK expert
- Work reviewed by course organiser

Project approach

- Convert HELM source to Moodle (Mathjax)
- Implement questions within STACK where possible
  - sensible randomisation
  - author written solutions
  - PRT, deployed variants, question notes
- Follow strict style guide and authoring guidelines
- Work reviewed by local STACK expert
- Work reviewed by course organiser
- Quizzes tested by UG student working on ASID

Challenges

- All authors new to STACK (16 PhD students in Edinburgh).
- Speed of work varied significantly.
- Varied use of STACK.
- No clear understanding of what *fully worked solutions* means.
- workbooks completed out of sequence.
- Repetitive feedback to different authors.

HELM to STACK

The alternating series test

An alternating series is a special type of series in which the sign changes from one term to the next. They have the form

$$a_1 - a_2 + a_3 - a_4 + \cdots$$

(in which each $a_i,\ i = 1, 2, 3, \ldots$ is a **positive** number)
Examples are:

(a) $1 - 1 + 1 - 1 + 1 \cdots$

(b) $\frac{1}{3} - \frac{2}{4} + \frac{3}{5} - \frac{4}{6} + \cdots$

(c) $1 - \frac{1}{2} + \frac{1}{3} - \frac{1}{4} + \cdots.$

For series of this type there is a simple criterion for convergence:

Key Point 6

The Alternating Series Test

The alternating series

$$a_1 - a_2 + a_3 - a_4 + \cdots$$

(in which each $a_i,\ i = 1, 2, 3, \ldots$ are **positive** numbers) is convergent if and only if

- the terms continually decrease:

$$a_1 > a_2 > a_3 > \ldots$$

- the terms decrease to zero:

$$a_p \to 0 \quad \text{as } p \text{ increases} \qquad (\text{mathematically } \lim_{p \to \infty} a_p = 0)$$

The alternating series test

An alternating series is a special type of series in which the sign changes from one term to the next. They have the form

$$a_1 - a_2 + a_3 - a_4 + \ldots$$

(in which $a_i,\ i = 1, 2, 3, \ldots$ is a **positive** number).

Examples are:

(a) $1 - 1 + 1 - 1 + \ldots$

(b) $\frac{1}{3} - \frac{2}{4} + \frac{3}{5} - \frac{4}{6} + \ldots$

(c) $1 - \frac{1}{2} + \frac{1}{3} - \frac{1}{4} + \ldots$

For this type of series there is a simple criterion for convergence:

Key Point:

The Alternating Series Test

The alternating series

$$a_1 - a_2 + a_3 - a_4 + \ldots$$

(in which $a_i,\ i = 1, 2, 3, \ldots$ are **positive** numbers) is convergent if both

- the terms continually decrease:

$$a_1 > a_2 > a_3 > \ldots$$

- and the terms decrease to zero:

$$a_p \to 0 \text{ as } p \text{ increases} \qquad \left(\text{mathematically } \lim_{p \to \infty} a_p = 0\right)$$

Note that if the series fails the alternating series test then it may still be convergent, you simply are not equipped to determine the outcome.

HELM to HTML

The alternating series test

An alternating series is a special type of series in which the sign changes from one term to the next. They have the form

$$a_1 - a_2 + a_3 - a_4 + \cdots$$

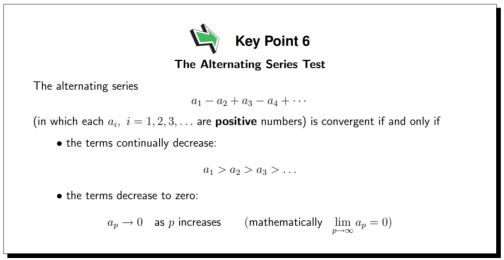
(in which each a_i, $i = 1, 2, 3, \ldots$ is a **positive** number)
Examples are:

(a) $1 - 1 + 1 - 1 + 1 \cdots$

(b) $\frac{1}{3} - \frac{2}{4} + \frac{3}{5} - \frac{4}{6} + \cdots$

(c) $1 - \frac{1}{2} + \frac{1}{3} - \frac{1}{4} + \cdots$.

For series of this type there is a simple criterion for convergence:



2.1 The alternating series test

An alternating series is a special type of series in which the sign changes from one term to the next. They have the form

$$a_1 - a_2 + a_3 - a_4 + \cdots$$

(in which each a_i, $i = 1, 2, 3, \ldots$ is a **positive** number)

Examples are:

1. $1 - 1 + 1 - 1 + 1 \cdots$
2. $\frac{1}{3} - \frac{2}{4} + \frac{3}{5} - \frac{4}{6} + \cdots$
3. $1 - \frac{1}{2} + \frac{1}{3} - \frac{1}{4} + \cdots$.

For series of this type there is a simple criterion for convergence:

Key Point 6

The Alternating Series Test

The alternating series

$$a_1 - a_2 + a_3 - a_4 + \cdots$$

(in which each a_i, $i = 1, 2, 3, \ldots$ are **positive** numbers) is convergent if and only if

- the terms continually decrease:

$$a_1 > a_2 > a_3 > \ldots$$

- the terms decrease to zero:

$$a_p \to 0 \quad \text{as} \quad p \quad \text{increases} \qquad \left(\text{mathematically} \quad \lim_{p \to \infty} a_p = 0\right)$$

Created by MASH at the University of Bath

HELM to STACK

Express as partial fractions $\dfrac{3x+1}{(x^2+x+10)(x-1)}$

Note that the quadratic factor cannot be factorised further. We have

$$\frac{3x+1}{(x^2+x+10)(x-1)} = \frac{Ax+B}{x^2+x+10} + \frac{C}{x-1}$$

First multiply both sides by $(x^2+x+10)(x-1)$:

Your solution

$3x+1 =$

Answer

$(Ax+B)(x-1)+C(x^2+x+10)$

Evaluate C by letting $x=1$:

Your solution

Answer

$4 = 12C$ so that $C = \dfrac{1}{3}$

Equate coefficients of x^2 and hence find A, and then substitute any other value for x (or equate coefficients of x) to find B:

Your solution

$A =$ $B =$

Answer

$-\frac{1}{3}$, $\frac{7}{3}$.

Finally express in partial fractions:

Your solution

Answer

$$\frac{3x+1}{(x^2+x+10)(x-1)} = \frac{-\frac{1}{3}x+\frac{7}{3}}{x^2+x+10} + \frac{\frac{1}{3}}{x-1} = \frac{7-x}{3(x^2+x+10)} + \frac{1}{3(x-1)}$$

Exercise

Express as partial fractions:

$$\frac{10x^2+6x+7}{(2x^2+12x+21)(4x-7)}$$

Note that the quadratic factor cannot be factorised further. We have

$$\frac{10x^2+6x+7}{(2x^2+12x+21)(4x-7)} = \frac{Ax+B}{2x^2+12x+21} + \frac{C}{4x-7}$$

First multiply both sides by $(2x^2+12x+21)(4x-7)$:

$10x^2+6x+7 =$

Evaluate C:

$C =$

Equate coefficients of x^2 and hence find A, and then substitute any other value for x (or equate coefficients of x) to find B:

$A =$

$B =$

Finally express in partial fractions:

$$\frac{10x^2+6x+7}{(2x^2+12x+21)(4x-7)} =$$

Check

Use of workbooks in Engineering / Chemistry students

Year 1 Engineering and Chemistry courses, $\sim$ 500 students.

Course Organiser: Dr David Quinn.

Use of workbooks in Engineering / Chemistry students

Year 1 Engineering and Chemistry courses, $\sim$ 500 students.

Course Organiser: Dr David Quinn.

- 1 HELM workbook per week (3 – 4 sections).
- served as first introduction to each topic.
- assessed by separate STACK quiz.
- video lectures used to expand on theory.
- traditional text used for 'written' problems and reducing screen time.

Success of workbooks

... the most valuable thing about this course are the workbooks in STACK. I have learnt a lot from the workbooks.

Working with STACK and being assessed regularly keeps me engaged even having to study remotely.

Student feedback rate HELM as the most valuable resource.

The 'check' button most of all!

I really like the STACK workbooks, they explain everything in a basic way.

The workbooks were a really good and clear way of laying out the maths with immediate questions to help develop understanding through examples.

Questions closer to exam questions could be put into the stack workbooks.

Future Plans

1. Review of year:
   - understand how students used HELM on STACK
   - identify and improve “weak” questions
   - identify where further questions are needed

Future Plans

1. Review of year:
   - understand how students used HELM on STACK
   - identify and improve “weak” questions
   - identify where further questions are needed
2. Review missing materials and try to “complete the set”;

Future Plans

1. Review of year:
   - understand how students used HELM on STACK
   - identify and improve “weak” questions
   - identify where further questions are needed
2. Review missing materials and try to “complete the set”;
3. Release all materials under CC (With the STACK source code).

Conclusions

Quality online learning materials require a team effort:

Conclusions

Quality online learning materials require a team effort:

- Online platform (e.g. STACK).

Conclusions

Quality online learning materials require a team effort:

- Online platform (e.g. STACK).
- Well-designed materials (e.g. HELM workbooks).

Conclusions

Quality online learning materials require a team effort:

- Online platform (e.g. STACK).
- Well-designed materials (e.g. HELM workbooks).
- Tested delivery format (e.g. FAC).

Conclusions

Quality online learning materials require a team effort:

- Online platform (e.g. STACK).
- Well-designed materials (e.g. HELM workbooks).
- Tested delivery format (e.g. FAC).
- Style guide/agreed delivery standard.

Conclusions

Quality online learning materials require a team effort:

- Online platform (e.g. STACK).
- Well-designed materials (e.g. HELM workbooks).
- Tested delivery format (e.g. FAC).
- Style guide/agreed delivery standard.

Quality materials require people to do the work!

Thanks to all the Edinburgh interns from the summer 2020.