e-Assessment of graph enumeration problems in Discrete Mathematics courses

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Student group

- Bachelor in Cyber Security/Programming (200 students)
- 1st year, 1st semester, 7.5 credits
- The **only** mathematics course during studies
- No strong prerequisites in mathematics
- No prior programming knowledge

Discrete mathematics

- Set theory, logic
- Combinatorics
- Number theory and some crypto
- **Graphs**
- (Very limited) Linear algebra

Graph theory

- Many relevant applications, fx network analysis, TSP
- Focus on algorithms (BFS, DFS, Dijkstra's, Prims)
- But: inductive proofs are hard to comprehend.
- Many students do equivalents of this:

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$ax^2 + bx + c = 0$$

Teacher challenge 1:
break this pattern

Dear students please start doing this:

- Start producing something authentical
- Find arguments for what you are doing is correct (essentially: why do we need a proof)
- Practice in constructing algorithms
- (essentially: working systematic and being aware of the system)

Graph enumeration problems

- Classical subject
- Example: List all trees on 5 vertices
- (usually **easy**): find **some**
- (usually **difficult**): find **all**
- Good training in **pure concepts** (e.g. what is a tree) and **isomorphy**: are two graphs the same or not?

Graphical Enumeration

1st Edition - May 28, 1973

☆☆☆☆☆ Write a review

Authors: Frank Harary, Edgar M. Palmer

eBook ISBN: 9781483273785

View on ScienceDirect

«Give an example»-type of problems

- Goldenberg, Mason, 2008. Shedding light on and with example spaces
- *Simply 'giving' examples and construction techniques is rarely sufficient for most learners. Most learners need to (re)construct examples in order to populate their example space*

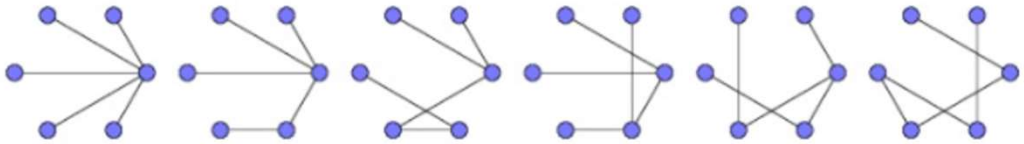
- Give one example
- Give two
- …
- Why are you done now?

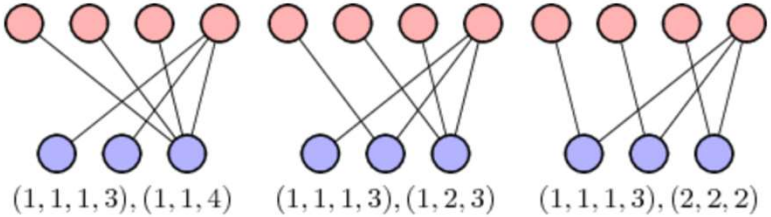
Teacher challenge 2: assess all this

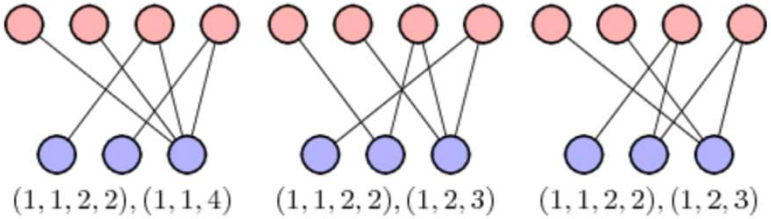
- 200 students, many online. Manual grading: not feasible.
  - Issue 1: too **many students**
  - Issue 2: too **demanding** compared to quadratic equations

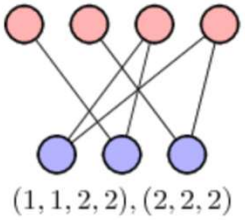
    (pictures found on stackexchange)

The 6 non-isomorphic trees are listed below.



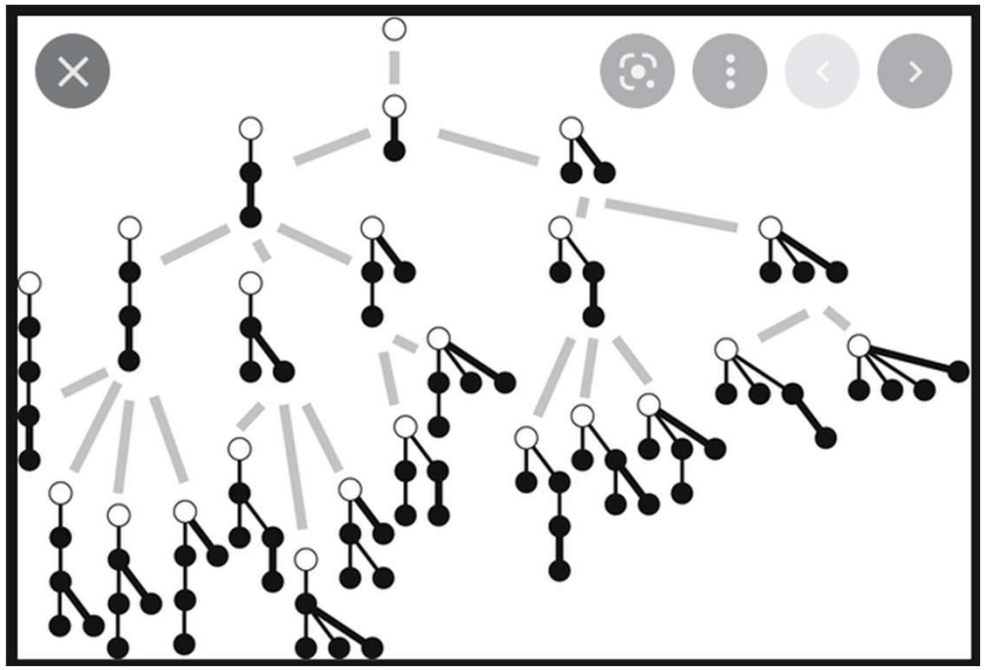






What we want at the end

- Make problem simpler (Polya. How to solve it, 1976)
- Look at how structure develops when a parameter grows
  (pic: Shin-ichi Nakano, Enc.Alg., 2016)
- This is something we focus at during interactions



Teacher challenge 3: automate it

- We need to input graphs!
- Three rep forms: matrix, list, drawing
- **Matrices**: a lot of entries! And permutations.

  (Q: find a disconnected graph on 5 nodes with 5 connected components)

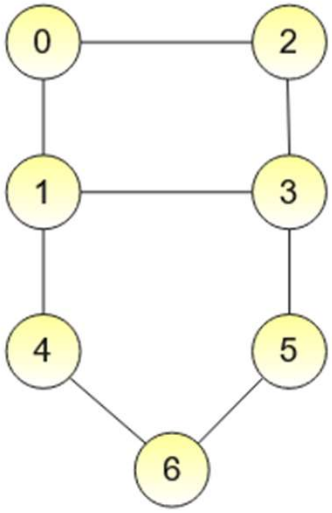
Angi ei nabomatrise for en ikke-sammenhengende graf med 5 noder og 5 komponenter. (En av de som du fant i deloppgave a, Grafteori 1) Når du skal velge rekkefølge på nodene, gjør det slik at flest mulig enere i matrisa havner nærmest mulig øvre venstre hjørne.

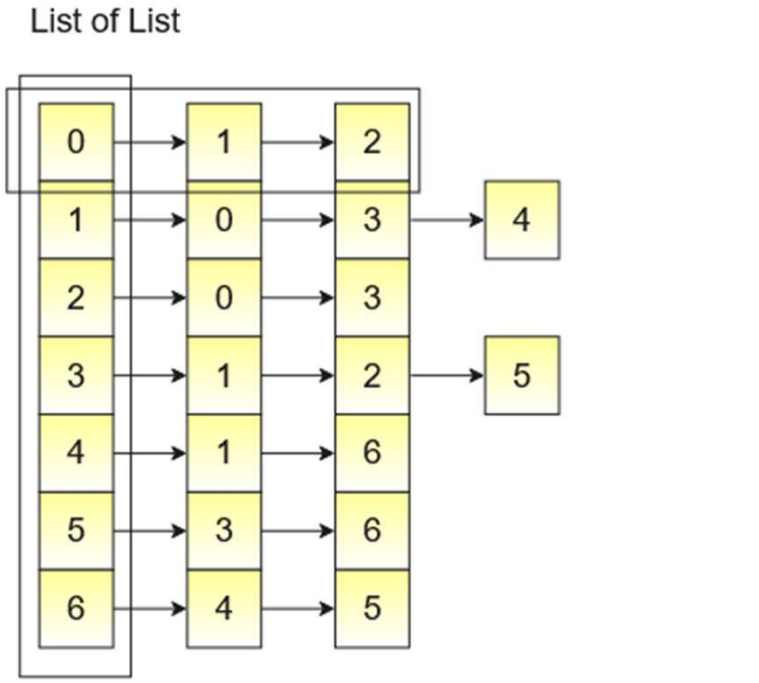
0,0	0,0	0,0	0,0	0,0
0,0	0,0	0,0	0,0	0,0
0,0	0,0	0,0	0,0	0,0
0,0	0,0	0,0	0,0	0,0
0,0	0,0	0,0	0,0	0,0

Teacher challenge 3: automate it

- We need to input graphs!
- Three rep forms: matrix, list, drawing
- **Lists**: Gets long as well. And permutations. Error-prone

(Pic: algotree.com)





Adjacency list representation for storing the graph

What exactly is student's issue?

- Is it under «tree» concept or under «matrix» concept?
- Is it under «being connected» concept or «list» concept?
  (remember 1st year, 1st semester)

- Student panel feedback:
  Many complain that difficult input makes them hate system

Teacher challenge 3: automate it

- We need to input graphs!
- Three rep forms: matrix, list, drawing
- **Drawing**: easy for small graphs. Natural rep for beginners.
- **But how to input into a CAS?**

Solution: CodeRunner for Moodle

- Solution: CodeRunner, a plugin for Moodle
- Citing https://moodle.org/plugins/qtype_coderunner

  A question type that allows question authors to set programming questions in which the student answer is code in some programming language, which is graded by running it.
- Surprisingly, there is an input type that allows to draw graphs (and outputs a list structure)
- There is a python package **networkx** that can do graphs

  https://networkx.org/
- So, we let students draw, get a CodeRunner structure, convert it to networkx and just write for example

```
print(nx.is_connected(G))
```