THE UNIVERSITY of EDINBURGH
School of Mathematics

Using online STACK assessment to teach complex analysis: a prototype course design?

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Overview

1 The course: context and design

2 How it went in practice

3 Future modifications

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Context

Honours Complex Variables

Richard Gratwick

First course in complex analysis: Holomorphic functions, singularities, Cauchy Integral Theorem, residue calculus, …

11 weeks	~250 students	Year 3

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Design

Classes

- Two 50-min lecture per week: interactive
- One 50-min workshop (tutorial) per week

Assessment

- 25% coursework: 5 hand-ins + skills assignments
- 75% final exam: timed, take-home, open-book

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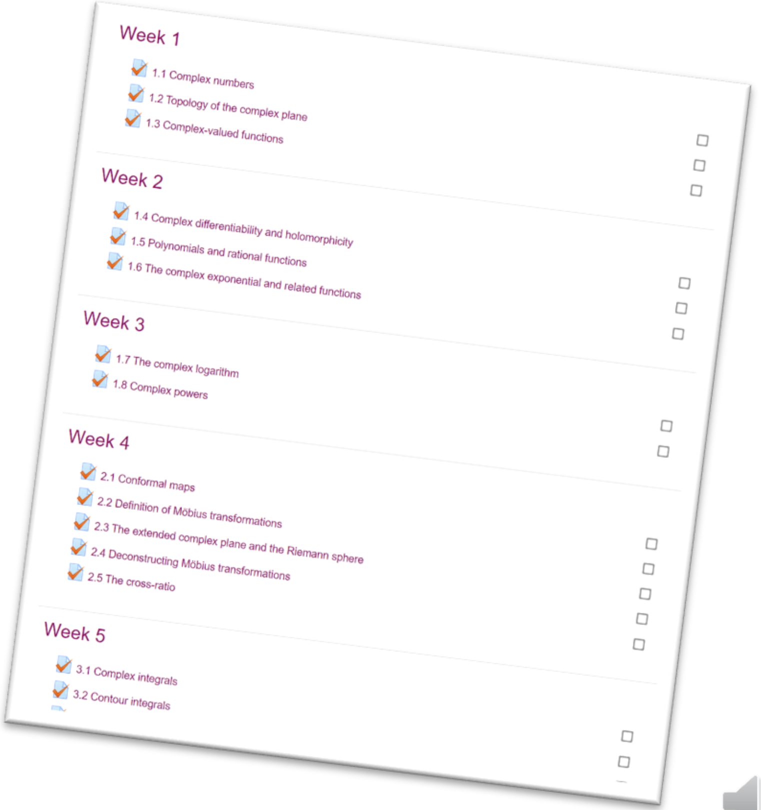
Workbooks

- Main reference
- One Moodle quiz for each section of the course notes
- Essentially used existing PDF notes in a multimedia and interactive format
- Students knew what was expected of them



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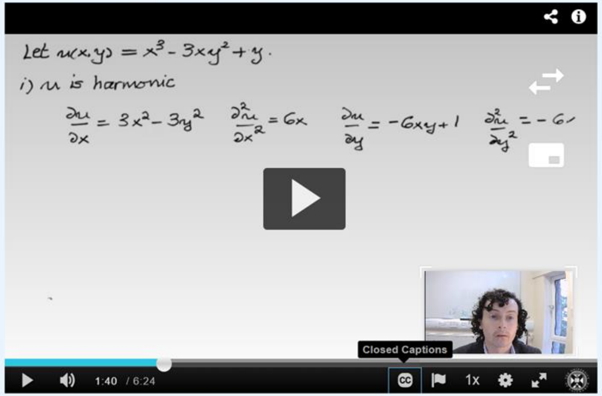
Definition 1.4.15

Let $U \subseteq \mathbb{R}^2$ be open, and let $u\colon U \to \mathbb{R}$ be harmonic. We say that a harmonic function $v\colon U \to \mathbb{R}$ is a ***harmonic conjugate of*** u if the complex-valued function $f = u + \mathrm{i}\, v$ is holomorphic on U.

Example 1.4.16

An example of finding a harmonic conjugate for a function.



Tidy STACK question tool | Question tests & deployed variant

✎ Exercise

Let $u\colon \mathbb{R}^2 \to \mathbb{R}$ be defined by $u(x, y) = -39\,x\,y^2 + 11\,y + 13\,x^3$. Prove that u is the real part of a holomorphic function $f\colon \mathbb{C} \to \mathbb{C}$ by constructing a harmonic conjugate $v\colon \mathbb{R}^2 \to \mathbb{R}$ such that $f = u + \mathrm{i}\, v$.

$v(x, y) =$ ______ .

Check

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Exercise

For each of the following subsets of $\mathbb{C}$, decide if it is open, closed, neither or both.

It may help you to draw a sketch of each subset.

Subset of $\mathbb{C}$	Classification
$D_2(0) \setminus D_1(0)$	(No answer given)
$D_1(\mathrm{i}) \cup D_1(0)$	
$D_1(\mathrm{i}) \cap D_1(0)$	(No answer given)
$D_2(0) \setminus \overline{D}_1(0)$	(No answer given)
$D_2'(0)$	(No answer given)
$\mathbb{C}$	(No answer given)
$D_1(0)$	(No answer given)
$\overline{D}_1(0) \setminus D_1(0)$	(No answer given)

(No answer given)
Open
Closed
Neither open nor closed
Open and closed

Check

1

Exercise

Evaluate the following contour integral.

$$\int_{C_{2\pi}(0)} \frac{\cos(z)}{z+\pi}\, dz = \underline{\qquad}.$$

Check

We can write the integrand f as

$$f(z) = \frac{g(z)}{z - z_0}$$

where $g(z) = \cos(z)$ is holomorphic inside and on the loop $C_{2\pi}(0)$, and $z_0 = -\pi$ lies inside $C_{2\pi}(0)$. Therefore the Cauchy Integral Formula implies that

$$\begin{aligned} \int_{C_{2\pi}(0)} f(z)\, dz &= \int_{C_{2\pi}(0)} \frac{g(z)}{z - z_0}\, dz \\ &= 2\,\mathrm{i}\,\pi\, g(z_0) \\ &= 2\,\mathrm{i}\,\pi \cos(-\pi) \\ &= -2\,\mathrm{i}\,\pi. \end{aligned}$$

A correct answer is $-2\,\mathrm{i}\,\pi$, which can be typed in as follows: `-(2*%i*%pi)`

Try another question like this one

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Distinctive things

- Using workbooks as the main resource in an advanced course
- Students get no credit for working through workbooks
- Interplay between different course activities

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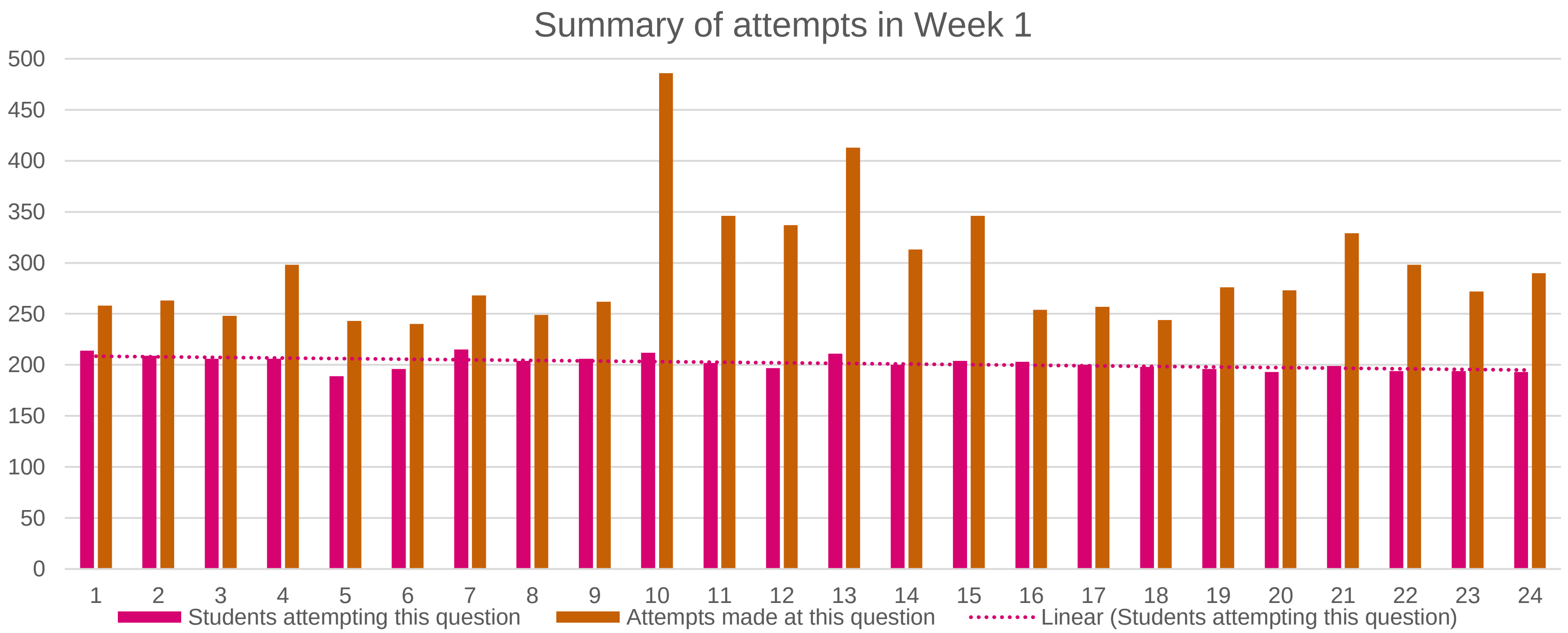
How did it go?



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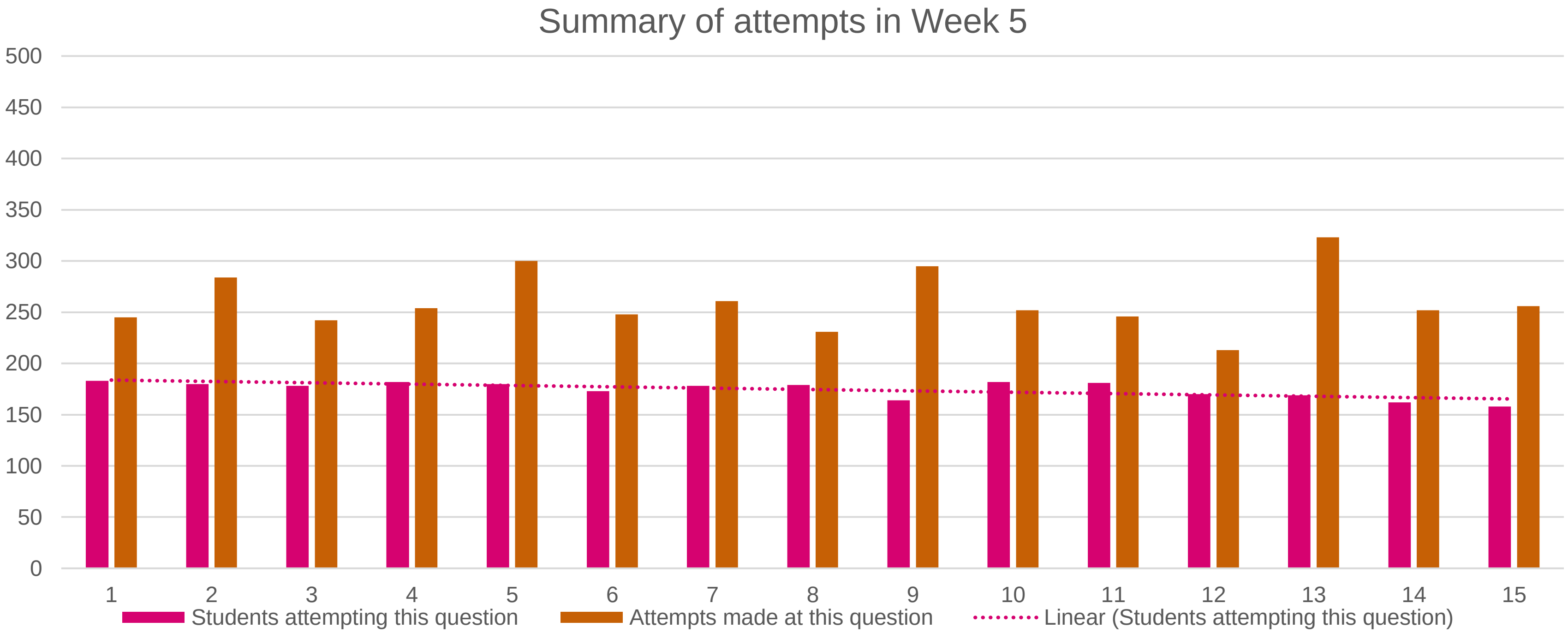
How did it go?



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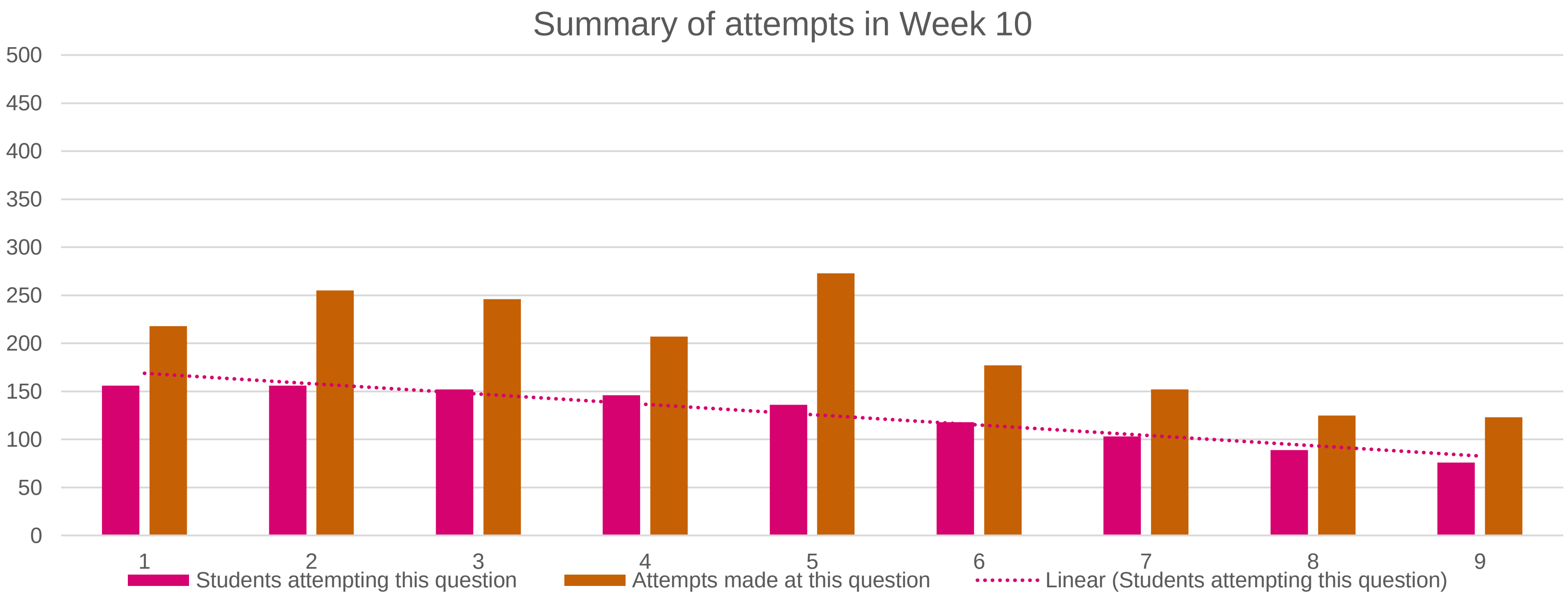
How did it go?



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How did it go?

"Recording short videos of examples and calculations allowed me to present these items "dynamically", rather than as plain text, but ***freed up time in synchronous lectures for me to engage with more conceptual high-level discussion*** *of the material. The Stack questions for self-assessment were largely based on exercises included in the previous version of the written notes, but* ***students engaged with them much more regularly when they were presented in the online workbooks*** *– my impression was that in previous years these written exercises were largely ignored."*

Richard Gratwick, Course Organiser

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How did it go?

*"Genuinely this course has been the **perfect mix of activities** for my learning, I'd go as far say to **the best organised course** I've taken in SoM, certainly this year anyway. The notes being delivered in stack are great and **much more engaging than a pdf** (the supplementary pdf is much more easy to navigate for finding Theorems etc. however), which actually **makes me do all the reading before lectures** so I gain so much more from them. Stack is good in part because of the **instant feedback** on most exercises which are immediately relevant to what you're learning, but also because it **breaks the material up well**. Stack being the main resource works in perfectly with the 2 lectures delivered a week and the tutorial. **SoM should considering delivering all courses in this fashion.**"*

A student

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Future modifications

- On-campus lectures
- Bring together all content in workbooks
- Support more effective groupwork

Thank you!

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