

DIAGNOSTIC TESTS IN NUMBAS

Don Shearman and Jim Pettigrew

Motivation/History

- Many students poorly prepared for university maths and stats study.
- Need for online tailored provision of refresher lessons in basic algebra.
- Need for an 'automated' system that diagnoses students' learning needs and recommends focus areas.

Motivation/History

- Many students poorly prepared for university maths and stats study.
- Need for online tailored provision of refresher lessons in basic algebra.
- Need for an 'automated' system that diagnoses students' learning needs and recommends focus areas.
- MESH adopted the Numbas Diagnostic Test algorithm.
- We received internal funding for development work.

Design and development

MESH designed and developed a tool based on an existing online refresher module in basic algebra (called 'Algebra 1').

Design and development

MESH designed and developed a tool based on an existing online refresher module in basic algebra (called 'Algebra 1').

The design has two components: a 'knowledge map' and a diagnostic tool (built in Numbas) drawing on the logic of the knowledge map.

Design and development

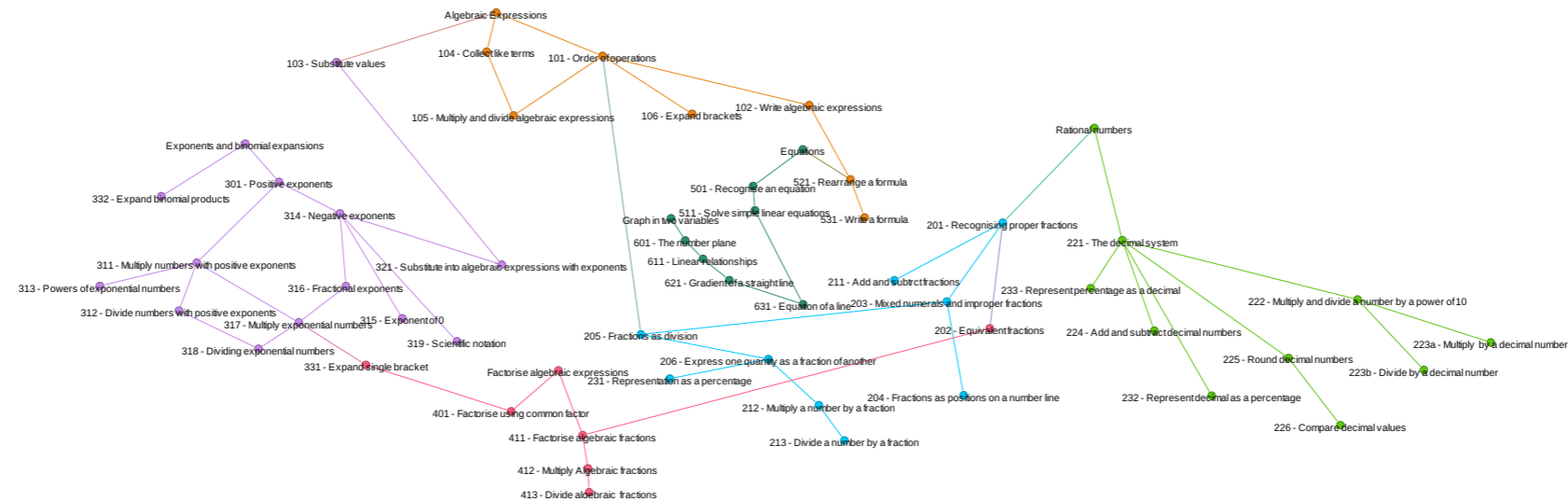
MESH designed and developed a tool based on an existing online refresher module in basic algebra (called 'Algebra 1').

The design has two components: a 'knowledge map' and a diagnostic tool (built in Numbas) drawing on the logic of the knowledge map.

The development involved refining the knowledge map and creating a set of Numbas questions.

DESIGN (KNOWLEDGE MAP)

The knowledge map has the form of a directed acyclic graph, where nodes represent topics (questions) and edges define the topic hierarchy.



DESIGN (DIAGNOSTIC)

The diagnostic tool was designed using Numbas.

- The Diagnostic exam type allows questions to be hierarchically arranged:
 - A wrong answer to a question causes the system to mark all harder questions on the same path as wrong.
 - A correct answer to a question causes the system to mark all easier questions on the same path as correct.

DEVELOPMENT

There are three key steps in the building process:

1. Matching questions to topics.
2. Creating learning outcomes and topics within them.
3. Linking topics according to the knowledge map (using 'depends on'/'leads to' directives).

DEVELOPMENT

There are three key steps in the building process:

1. Matching questions to topics.
2. Creating learning outcomes and topics within them.
3. Linking topics according to the knowledge map (using 'depends on'/'leads to' directives).

This development process was recursive in the sense that matching questions to topics led to some refinement of the knowledge map.

The Numbas authoring interface allows for easy topic linking:

Topics Learning objectives Diagnostic algorithm

Topics

- 101 - Order of operations
- 102 - Write an algebraic expression
- 103 - Substitute values
- 104 - Collect like terms
- 105 - Multiply and divide algebraic expressions
- 106 - Expand brackets
- 201 - Recognise proper fractions
- 202 - Equivalent fractions
- 203 - Mixed numerals and improper fractions
- 204 - Fractions as positions on a number line
- 205 - Fractions as division
- 206 - Express one quantity as a fraction of another
- 211 - Add and subtract fractions
- 212 - Multiply a number by a fraction
- 213 - Divide a number by a fraction

Name
101 - Order of operations

Description

← Depends on

→ Leads to

102 - Write an algebraic expression ✕ 105 - Multiply and divide algebraic expressions ✕ 106 - Expand brackets ✕ 205 - Fractions as division ✕

Learning objectives

Algebraic expressions ✕

Questions in this topic

Implementation

The tool was offered to students in February this year. To date (22/6/2023), it has been attempted 715 times by 521 unique students.

- Some attempts were incomplete, with some having no questions answered.

Analysis

Using attempt data from our LTI server, we have begun evaluating the diagnostic tool. Our main aim is to ensure that the knowledge map is valid and the questions based on it are properly constructed and targeted.

This is a work in progress (as determining a robust method of validation has required exploration of the research base).

Our analysis has unearthed a few unanticipated considerations:

- We have gained a deeper understanding of students' behaviour in interacting with the tool — for example:
 - some quit once they got a question (or a few questions) wrong.
 - some looked ahead and then quit.
 - many complete attempts took significant time (> 45 mins).
- There is scope for feature improvements of the tool — for example:
 - question sequence (perhaps a 'binary' selection?).
 - optimisation of efficiency of attempts: certain questions, if answered correctly, eliminate a large set of questions 'above' it.

DATA EXTRACTION, RESTRUCTURING, CLEANING

- Extraction via a JSON file.
- Restructuring and cleaning in R (dataframe with 'null' cases removed).

```
Object
  resource: Object
  pk: 386
  title: "Diagnostic tool for Algebra 1"
  attempts: Array[678]
    > 0: Object
    > 1: Object
    > 2: Object
    > 3: Object
    > 4: Object
    > 5: Object
    > 6: Object
      attempt: 31662
      resource: Object
      exam: 534
      user: Object
      start_time: 1682415294.43
      end_time: null
      completion_status: "incomplete"
      scaled_score: 0.3
      raw_score: 0.3
      scores: Array[28]

      broken: false
      remarked_parts: Array[0]
      scorm: Object
      suspend_data: Object
    > 7: Object
    > 8: Object
```

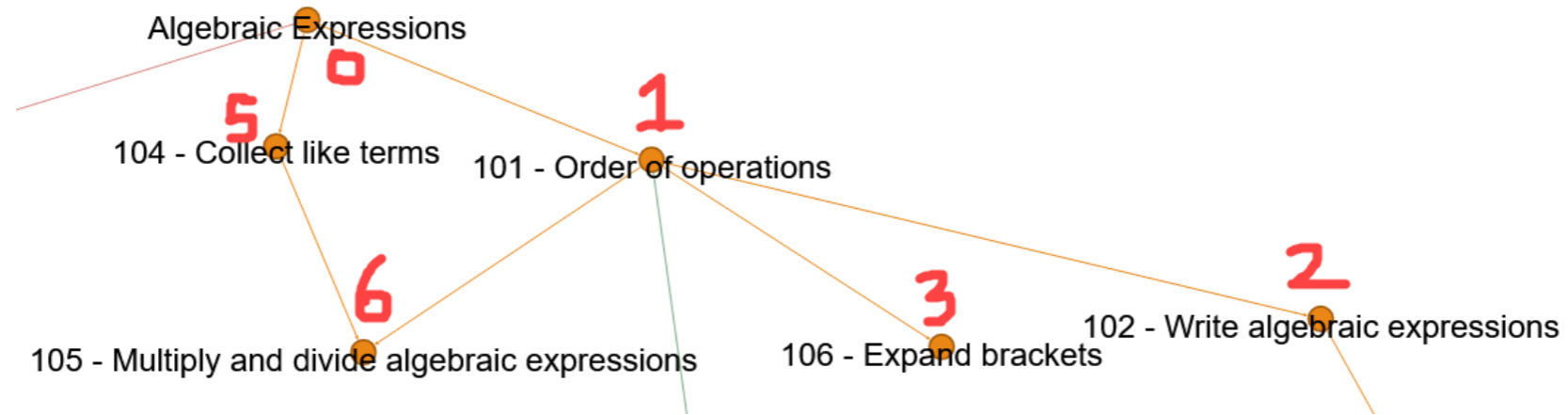
USE OF IGRAPH PACKAGE IN R TO REPRESENT THE KNOWLEDGE MAP

Knowledge map imported into R using the igraph package.

Each item is associated with 'out' and 'in' nodes — if a student gets a question (node) wrong, then the implication is that they will get all 'out' nodes wrong (the 'zeros sink'); if they get the question right, the implication is that they will get all 'in' nodes right (the 'ones float').

```
[1] 0 ->1 1 ->2 1 ->3 0 ->4 0 ->5 5 ->6 1 ->6 7 ->8 8 ->9 8 ->10 10->11 10->12 1 ->12
[14] 12->13 13->14 14->15 13->16 8 ->17 7 ->18 18->19 19->20 19->21 18->22 18->23 23->24 18->25
[27] 18->26 27->28 28->29 29->30 29->31 28->32 32->33 32->34 34->35 29->35 35->36 30->36 32->37
[40] 32->38 4 ->38 35->39 40->41 39->41 40->42 9 ->42 42->43 43->44 27->45 46->47 47->48 46->49
[53] 2 ->49 49->50 51->52 52->53 53->54 54->55 48->55
```


[1] 0 ->1 1 ->2 1 ->3 0 ->4 0 ->5 5 ->6 1 ->6 7 ->8 8 ->9 8 ->10 10->11 10->12 1 ->12
 [14] 12->13 13->14 14->15 13->16 8 ->17 7 ->18 18->19 19->20 19->21 18->22 18->23 23->24 18->25
 [27] 18->26 27->28 28->29 29->30 29->31 28->32 32->33 32->34 34->35 29->35 35->36 30->36 32->37
 [40] 32->38 4 ->38 35->39 40->41 39->41 40->42 9 ->42 42->43 43->44 27->45 46->47 47->48 46->49
 [53] 2 ->49 49->50 51->52 52->53 53->54 54->55 48->55



IMPLIED SCORING

All non-NA raw question responses were converted to 0 or 1:

- $\text{response} < 1 \rightarrow \text{response} = 0$
- $\text{response} = 1 \rightarrow \text{response} = 1$

Knowing the sequence of a student's question responses, we have applied the above knowledge map logic to generate implied scores for all questions in the tool.

This has enabled the creation of 359 'fully implied scored' attempts (complete cases).

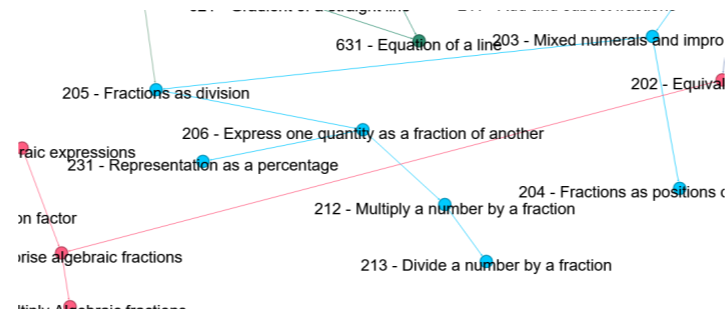
RASCH MODELLING

We have used Rasch modelling of raw and implied response sets (on complete cases only).

And compared the item difficulty rankings to glean information about questions and their place within the knowledge map hierarchy.

	1	3	4	5	6	9	12	13	14	17	18	19
1	Question Number	item	N (raw)	M (raw)	xsi.item (raw)	RawRank	N (implied)	M (implied)	xsi.item (implied)	ImpliedRank	RankDifference Signed	RankDifference Unsigned
2	11	Q11_score	62	0.20967742	0.295748491	39	359	0.86350975	-2.875035341	5	-34	34
3	46	Q46_score	112	0.83928571	-1.17161844	16	359	0.26183844	1.648897408	42	26	26
4	31	Q31_score	193	0.9119171	-1.98534228	6	359	0.4902507	0.039285652	31	25	25
5	22	Q22_score	222	0.92342342	-2.48751312	1	359	0.57103064	-0.482518206	25	24	24
6	12	Q12_score	95	0.34736842	-0.1710702	30	359	0.82729805	-2.46509587	7	-23	23
7	9	Q9_score	49	0.30612245	-0.7416318	23	359	0.90529248	-3.468403653	4	-19	19
8	37	Q37_score	24	0.75	-0.20803235	28	359	0.19220056	2.282215311	46	18	18
9	39	Q39_score	72	0.81944444	-0.12853028	31	359	0.1643454	2.578939434	49	18	18
10	16	Q16_score	55	0.45454545	-0.99499117	19	359	0.91643454	-3.661885033	3	-16	16
11	17	Q17_score	75	0.49333333	-0.88529527	20	359	0.8356546	-2.553674552	6	-14	14

Q11 (205 - Fractions as division) was found to have the greatest difference in item difficult ranking (raw vs implied). For many cases, Q11 has been assigned an implied score of 1.



VertexIDigraph	VertexNameIgraph	QuestionNumbersDiagnosticTool	outVertices	inVertices
1	101 - Order of operations		c(12, 2, 3, 6, 13, 49, 14, 16, 50, 15)	NA
10	203 - Mixed numerals and improper fractions		c(11, 12, 13, 14, 16, 15)	8
11	204 - Fractions as positions on a number line		NA	c(10, 8)
12	205 - Fractions as division		c(13, 14, 16, 15)	c(1, 10, 8)
13	206 - Express one quantity as a fraction of another		c(14, 16, 15)	c(12, 1, 10, 8)
14	212 - Multiply a number by a fraction		15	c(13, 12, 1, 10, 8)
15	213 - Divide a number by a fraction		NA	c(14, 13, 12, 1, 10, 8)
16	231 - Representation as a percentage		NA	c(13, 12, 1, 10, 8)
17	211 - Add and subtrct fractions		NA	8
18	221 - The decimal system		c(19, 22, 23, 25, 26, 20, 21, 24)	NA

This might not be appropriate as Q11 contains a part that is relatively 'hard'.

Q11: 205 - Fractions as division

Q15: 213 - Divide a number by a fraction

'Easy'

'Hard'

Q11: 205 - Fractions as division

Q15: 213 - Divide a number by a fraction

Complete the following without using a calculator.

a)

$$\frac{1}{3} \div \frac{1}{5}$$

Give your answer as a fraction (proper or improper).

Reduce your answer to lowest terms.

b)

$$1 - \frac{1 - \frac{6}{7}}{\frac{3}{4}}$$

Reduce your answer to lowest terms.

Complete the following without using a calculator.

a)

$$12 \div \frac{1}{8}$$

b)

$$\frac{3}{7} \div 5$$

Reduce your answer to lowest terms.